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Consistent completion of incomplete judgments in decision making using AHP

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ABSTRACT

Decision making (DM) processes are becoming increasingly complex. The reasons are manifold. DM usually involves many aspects; some are purely technical, while others are subjective and derived from social, political, and environmental factors, among others. As a result, they involve items that are not easily comparable under the same units of measurement. Problems are made even more complex by the fact that current governance processes tend to involve all the stakeholders in the DM process.

In this paper we consider the AHP methodology (analytic hierarchy process), which is used to build consistent aggregate results from data provided by decision makers. As some of the actors involved may not be completely familiar with all the criteria under consideration, it is common that the body of opinion, expressed in terms of pairwise comparison, is incomplete. To overcome this weakness, we propose a framework that enables users to provide data on their preferences in a partial and/or incomplete way and at different times. This article is an advance towards a dynamic model of AHP. The authors have addressed the problem of adding a new criterion or deleting obsolete criteria. Here, we address the consistent completion of a reciprocal matrix as a mechanism to obtain a consistent body of opinion issued in an incomplete manner by a specific actor. This feature is incorporated into a process of linearization previously introduced by the authors, which is concisely presented. Finally, we provide an application for leakage control in a water supply company. The adoption of suitable control leakage policies in water supply is a problem of enormous interest in the water industry, particularly in urban hydraulics.

1. Introduction

The alternatives may be varied in decision making (DM) processes, and the decision consists in choosing the most desirable alternative after considering a set of criteria. The decision process can be complicated for several reasons. One of these reasons arises from the fact that criteria are not often comparable using the same unit of measurement. Also, there is a tendency in current governance processes to involve all the stakeholders in the DM process. For these reasons the process of decision making can be very complex and so adequate tools are necessary to support the process.

AHP (analytic hierarchy process) was introduced by Saaty [1–3] and with appropriate modifications the process can be used to integrate all of these aspects. Its hierarchical structure is an effective framework for DM and organizes the problem in terms of objectives, criteria, and alternatives. Also, the evaluation of criteria and alternatives in pairs and the subsequent

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construction of comparison matrices have been shown to be effective mechanisms for the joint treatment of tangible and intangible objectives. The use of appropriate mathematical techniques enables the prioritization of heterogeneous – and often very different – elements. This is crucial in decision-making. We present the basic elements of AHP in Section 2.

There are several problems associated with this methodology. The main problem is the possible lack of consistency in comparison matrices, as comparative judgments are subjective since they are issued by experts and/or other actors in the decision process. These matrices should accommodate a chosen scale (see [4–7], including, of course, the work by Saaty). Consequently, it is predictable – even reasonable – that the issued global opinion body lacks a minimum consistency, which is essential for the prioritization to be meaningful, reliable, and consequently, conducive to a sound decision. The literature contains numerous mechanisms to improve consistency, [8–17] among many others, and any attempt to improve consistency results in a better quality decision [6]. Among the methods to improve consistency, we present in Section 3 the linearization method introduced by the authors [16] on which the main contribution of this article is built.

A major problem is caused by the growing necessity for all the actors to be involved in decision processes. This leads to a couple of challenges. Firstly, the design of appropriate mechanisms for achieving consensus on a final decision that integrates the different points of view, possibly conflicting, of the various actors. This is one of the challenges to which most effort is currently being devoted; see among others [18–22]. However, as a precondition, some actors may not be completely familiar with one or more of the elements about which they have to issue their judgment or opinion. The authors have addressed this issue in [23] for a specific scenario: the addition or deletion of a criterion. In leakage control, for example, only economic aspects have so far been considered. However, environmental concerns are becoming important, and more recently, social elements have begun to play an important role in decision-making policy on leakage control. It is natural that some of the actors involved are not familiar enough with all the issues to make appropriate comparison judgments. In this paper we address this scenario: that of stakeholders being consulted when they are not familiar with the effects that some elements may have in the problem. As a result, it is difficult to gather complete information about the preferences of such a decision maker at a given moment. It seems reasonable to allow such an actor to express their preferences several times at his or her own convenience. Meanwhile, partial results based on partial preference data may be generated from data collected at various times—and this data may eventually be consolidated when the information is complete.

Several authors [24,25] have addressed the problem of producing preference data generated from incomplete information using various techniques that mainly involve optimization applied to different objective functions. For example, in [25] an approximation to the priority vector is obtained from an incomplete judgment. In Section 4 we provide a full matrix termination mechanism for an incomplete comparison matrix produced by an actor. This mechanism uses an algebraic method (instead of optimization processes) to minimize a distance in a matrix set (see [23, Section 2.2] for a justification of this metric). As a result it is efficient, robust, and easy to use.

To conclude the paper, in Section 5 we present the case of a judgment made by an actor from a water supply system in relation to leakage management policies. We present some conclusions and comments on specific elements for future work in Section 6.

2. AHP basics

The AHP developed by Saaty [1] formalizes the intuitive understanding of complex problems by building a hierarchical model.

The purpose of the method is to allow the actor involved to visually structure a multicriteria problem in a hierarchical manner. This hierarchy consists of three levels: the highest level contains the goal, the middle level contains the criteria, and the lowest level presents alternatives. Once the hierarchical model is constructed, comparisons are made between pairs of criteria and also between pairs of alternatives for each criterion. The process typically concludes by providing a summary of results through a process of aggregation.

The entire process is based on the fact that it enables the assignation of numerical values to the judgments given by the actor, making it possible to measure how each element contributes to the level of the hierarchy that is immediately above. Use is made of a specific scale for these comparisons in terms of preference or importance. We use here the scale developed by Saaty [1], with the possibility of including intermediate numerical (decimal) values in the scale to model hesitation between two adjacent judgments [26].

In the first step, the expert makes a comparison between pairs of criteria. Based on the scale of values, a comparison matrix of criteria is built. It is a square matrix of order n, $A = [a_{ij}]$, $1 \le i, j \le n$, where n is the number of criteria considered. The element a_{ij} represents the comparison between element i and element j. Subsequently, a similar exercise comparing alternatives for each criterion is performed, thus building comparison matrices of alternatives.

We recall here the main facts about this type of matrix. Consider a real matrix of size $n \times n$. A is *positive* if $a_{ij} > 0$ for all i, j, A is *homogeneous* if A is positive and $a_{ii} = 1$ for all i, A is *reciprocal* if A is positive and $a_{ij} = 1/a_{ji}$, for all i, j. These are typical properties of comparison matrices commonly found in AHP. In addition, A is *consistent* if A is positive and $a_{ik} = a_{ij}a_{jk}$, for all i, j, k. We shall always consider vectors of $\mathbb{R}^n$ as column vectors. Among the various characterizations of consistent matrices, let us recall the following:

Proposition 1 (*Theorem 1 of [17]*)**.** *A positive matrix A is consistent if and only if there is a vector $\mathbf{x}$ in $\mathbb{R}^n$ such that $A = \mathbf{x}J(\mathbf{x})^T$, where J is the map that associates a positive matrix $A = [a_{ij}]$ with the matrix whose entry (i, j) is $1/a_{ij}$.*