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Efficient construction of multi-block volumetric spline parameterization by discrete mask method

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ABSTRACT

Isogeometric analysis attempts to unify the mathematical languages in design and analysis to realize the seamless integration of CAD and CAE. In three-dimensional isogeometric analysis, parametric volume is employed as the computational domain for a given set of boundary information. In this paper, we propose a discrete mask method for the efficient construction of multi-block volumetric parameterization based on a set of given boundary spline surfaces. Given the block-partition information of a model, the interior control points can be obtained efficiently by solving a sparse linear system. The existence and uniqueness of the solution for the linear system are also proved. After performing a pre-process on the non-compatible boundary surfaces, the discrete Coons mask is generalized to a unified form, which provides more choices for the construction of inner control points. The proposed method is not only suitable for the multi-block case with C^1 continuity, but can also be used for the case in which the boundary surfaces are of different degrees, and with different number of control points and knot vectors. Several examples are presented to illustrate the effectiveness of the proposed method.

1. Introduction

Isogeometric analysis, which offers the possibility of seamless integration between CAD and CAE, is a new computational method proposed by Hughes et al. [1]. The approach employs spline representation both for the geometry and for the physical solutions, which avoids costly forth and back transformations. On the other hand, it reduces the degree of freedom to describe the geometry, which is of particular interest for shape optimization. Parameterization of the computational domain in isogeometric analysis, which corresponds to the mesh generation in FEA [2,3], also has some impact on the analysis result and efficiency [4,5]. As it is pointed by Cottrell et al. [6], one of the most significant challenges towards isogeometric analysis is the construction of an analysis-suitable volumetric parameterization based on the given CAD boundary representation.

There are works on volumetric parameterization from the viewpoint of graphics applications, and there is related work that requires a triangulated surface as input data [7–9]. In the context of isogeometric analysis, there are works on trivariate volumetric parameterization constructed from CAD boundary information with spline representation [10–13]. Moreover, existing work usually requires performing an optimization to achieve a high-quality volumetric parameterization, which

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is computationally expensive. In this paper, we propose a discrete mask method for the efficient construction of multi-block volumetric parameterization based on a set of given boundary spline surfaces, which can be employed as an initial construction of an analysis-suitable volumetric parameterization. Our main contributions are:

- An efficient framework based on discrete mask representation is proposed to generate a multi-block volumetric parameterization from given CAD boundary data, which can be obtained by solving a sparse linear system. The existence and uniqueness of the solution for the linear system are proved.
- Some special masks, such as harmonic mask and Dirichlet mask, are proposed for constructing volumetric parameterization with uniform and orthogonal isoparametric structures.
- Several volumetric modeling examples are presented to illustrate the effectiveness of the proposed method.

The rest of the paper is organized as follows. Section 2 reviews the related work in volumetric parameterization. Section 3 presents a single-block volumetric spline construction from non-compatible boundary surfaces by using the discrete mask method. The existence and uniqueness of the solution are also proved in this section. Multi-block volume parameterization with C^1 continuity is studied in Section 4. Some modeling examples are also presented in this section. Finally, we conclude this paper and outline future works in Section 5.

2. Related work

As far as we know, there are only a few works on the parameterization of computational domains from the viewpoint of isogeometric applications. Martin et al. [9] proposed a method to fit a genus-0 triangular mesh by using B-spline volume parameterization, based on discrete volumetric harmonic functions; this can be used to build computational domains for 3D IGA problems. A variational approach for constructing NURBS parameterization of swept volumes is proposed by Aigner et al. [14]. Cohen et al. [15] proposed the concept of *analysis-aware modeling*, in which the parameters of CAD models have to be selected to facilitate isogeometric analysis. Escobar et al. proposed a method to construct a trivariate T-spline volume of complex genus-zero solids for isogeometric application by using an adaptive tetrahedral meshing and mesh untangling technique [8]. Zhang et al. proposed a robust and efficient algorithm to construct injective solid T-splines for genus-zero geometry from a boundary triangulation [7]. Based on the Morse theory, a volumetric parameterization method for mesh model with arbitrary topology is proposed in [16]. The above proposed methods demand a surface triangulation as input data. For CAD model boundary with spline representation, r-refinement method for generating optimal analysis-aware parameterization of computational domain is proposed based on shape optimization method [4,5]. However, it only works for specified analysis problems. Pettersen and Skytt proposed the spline volume faring method to obtain high-quality volume parameterization for isogeometric applications [13]. Zhang et al. studied the construction of conformal solid T-spline from boundary T-spline representation using octree structure and boundary offset [11]. In this paper, we propose a discrete mask method for efficient construction of multi-block volume parameterization based on a given set of given boundary surfaces, which can be employed as an initial construction for the optimization methods proposed in [10,12] to obtain analysis-suitable volume parameterization of computational domain.

3. Single-block volumetric spline construction from non-compatible boundary surfaces

In this section, we will propose the discrete Coons volume construction method for non-compatible boundary surfaces and then extend the discrete Coons mask to general mask method.

3.1. Pre-processing for compatible boundary surfaces

In [4,5], the discrete Coons method is proposed to construct B-spline volume by using a linear combination of boundary control points. In this approach, the opposite B-spline surfaces should be compatible, that is, the opposite B-spline surfaces should have the same degree, knot vectors and number of control points [17]. However, in practice, such requirements are usually not satisfied. Hence, some pro-processing operation must be performed for the given opposite B-spline surfaces according to the following operation procedure:

1. make the given opposite B-spline surfaces to have consistent parametric direction;
2. perform degree elevation to ensure the surfaces are of the same degree;
3. knot vectors are scaled to lie in the interval $[0, 1]$;
4. perform knot insertion such that the surfaces are having the same number of control points.

When all the opposite surfaces on the boundary are compatible, the discrete Coons method can be employed to construct the single-block B-spline volumes, which will be reviewed in the following subsection.

3.2. Discrete version of Coons volume

Suppose that the given boundary surfaces are B-spline surfaces, the opposite boundary B-spline surfaces are compatible if they have the same degree, number of control points and knot vectors. If the boundary control points are given as shown