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A smoothing Broyden-like method with a nonmonotone derivative-free line search for nonlinear complementarity problems

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ABSTRACT

By using the CHKS-function, we propose a smoothing Broyden-like method for general nonlinear complementarity problems (NCPs). The method is based on the smoothing equation for which we consider the smoothing parameter as an independent variable, and makes use of a new nonmonotone derivative-free line search rule. Under suitable assumptions, we show that the iteration sequence generated by the proposed algorithm converges globally and superlinearly. Furthermore, the algorithm has local quadratic convergence under mild assumptions. Some numerical results are reported, which show that the algorithm is quite effective.

1. Introduction

The nonlinear complementarity problem, denoted by (NCP) [1–3], is to find a vector $x \in R^n$, such that

$$x \geq 0,\ F(x) \geq 0, \quad x^T F(x) = 0, \tag{1.1}$$

where $F : R^n \to R^n$ are continuously differentiable functions. The nonlinear complementarity problem has many important applications in operations research [1,4], engineering design [3,5] and economic equilibrium [5,6], and many numerical methods are developed to solve nonlinear complementarity problems.

In the last few years, there has been strong interests in smoothing method for solving the NCPs [7–11] partially due to their superior numerical performance. The ideal of smoothing method uses a smoothness function to reformulate the problem as a family of parameterized smooth equations, then solves the smooth equations approximately by using Newton method at each iteration. In [9], by exploiting a Jacobian consistency property for smoothing functions, Chen et al. designed the first globally and superlinearly convergent smoothing algorithm in absence of strict complementarity. Qi et al. [10] treated the smoothing parameter as a free variable, and proposed a class of new smoothing Newton methods for NCP and box constrained variational inequalities. Their algorithm solved only one linear system of equations and performed only one line search per iteration, and was proved to possess fast local convergence under a nonsingularity assumption. However, it is expensive for us to solve the smoothing equation, because one has to do lots of computations of the Jacobian at each iteration. This drawback motivated the development of quasi-Newton method for NCP. In [12,13], Ma et al. proposed smoothing

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Broyden-like methods for solving NCP. The methods were based on the smooth approximation Fischer–Burmeister function and made use of the derivative-free line search rule of Li in [14]. They proved that the algorithm was both globally and superlinearly convergent under suitable conditions. Recently, Chen and Ma [15] presented smoothing Broyden-like methods for solving P_0-NCP. The presented algorithm was based on the smoothing symmetrically perturbed minimum function $\phi(a, b) = \min\{a, b\}$ and made use of the derivative-free line search rule of Li et al. [16]. The global linear convergence and local superlinear/quadratic convergence of the proposed algorithm were discussed in [15]. Finally, they tested some numerical experiments to verify the method's validity.

Inspired by the above mentioned, we present a new Broyden-like algorithm for nonlinear complementarity problem. The method is based on the smoothing equation which considers the smoothing parameter as an independent variable. In order to achieve better numerical results, we adopt a new nonmonotone derivative-free line search rule in our method. We will show that the algorithm possesses some nice global and local convergence properties under mild assumptions.

The rest of this paper is organized as follows. In Section 2, we first present some preliminary results, then we establish a nonmonotone smoothing Broyden-like algorithm for solving NCP and show that the proposed algorithm is well defined. In Sections 3 and 4, we establish the global and local convergence of the proposed algorithm, Some numerical results are reported in Section 5. Finally, the conclusions are given in Section 6.

The following notions will be used throughout this paper. All vectors are column vectors, the subscript T denotes transpose, R^n (respectively, R) denotes the space of n-dimensional real column vectors (respectively, real numbers), R_+ (respectively, R_{++}) denotes the non-negative (respectively, positive) orthant in $\mathbb{R}$. For convenience, we write $(u^T, v^T)^T$ as (u, v) for any vectors u and v. We define $N := \{1, 2, \ldots, n\}$. For any vector $u \in R^n$, we denote by $\mathrm{diag}\{u_i : i \in N\}$ the diagonal matrix whose ith diagonal element is u_i and by $\mathrm{vec}\{u_i : i \in N\}$ the vector u. The symbol $\|\cdot\|$ denotes the Euclidean norm or the subordinate matrix norm. For a continuously differentiable mapping $G : R^n \to R^m$, we denote its Jacobian at a point $x \in R^n$ by $G'(x)$, whereas $\nabla G(x)$ denotes the transposed Jacobian. For any $\alpha, \beta \in R_{++}$, $\alpha = O(\beta)$ (respectively, $\alpha = o(\beta)$) means α/β is uniformly bounded (respectively, tends to zero) as $\beta \to 0$.

2. Preliminaries and algorithm

In this paper, our smoothing Broyden-like method is based on the smoothing function $\phi : R^3 \to R$ denoted by

$$\phi(\mu, a, b) := a + b - \sqrt{(a-b)^2 + 4\mu^2}, \quad \forall(\mu, a, b) \in R^3, \tag{2.1}$$

which was introduced by Chen–Harker–Kanzow–Smale [7]. The following lemma gives two simple properties of the smoothing function ϕ defined by (2.1). Its proof is obvious.

Lemma 2.1. *Let $(\mu, a, b) \in R^3$ and $\phi(\mu, a, b)$ be defined by (2.1). Then*

(1) $\phi(0, a, b) = 0 \Longleftrightarrow a \geq 0, b \geq 0, ab = 0.$
(2) *$\phi(\mu, a, b)$ is continuously differentiable for all points in R^3 different from $(0, c, c)$ for arbitrary $c \in R$. In particular, if $\mu > 0$, $\phi(\mu, a, b)$ is continuously differentiable for arbitrary $(a, b) \in R^2$.*

Let $z := (\mu, x) \in R_+ \times R^n$ and

$$H(z) := H(\mu, x) := \begin{pmatrix} e^{\mu} - 1 \\ \Phi(\mu, x) \end{pmatrix}, \tag{2.2}$$

where

$$\Phi(\mu, x) := \begin{pmatrix} \phi(\mu, x_1, F_1(x)) \\ \vdots \\ \phi(\mu, x_n, F_n(x)) \end{pmatrix}. \tag{2.3}$$

Thus by Lemma 2.1, we know that (1.1) is equivalent to the following equation:

$$H(z) = 0, \tag{2.4}$$

in the sense that their solution sets are coincident.

By simple calculation, it is not difficult to see that $H(\cdot)$ is continuously differentiable at any $z := (\mu, x) \in R_{++} \times R^n$ with its Jacobian

$$H'(z) = \begin{pmatrix} e^{\mu} & 0 \\ v(z) & D_1(z) + D_2(z)F'(x) \end{pmatrix}, \tag{2.5}$$

where

$$v(z) := vec\{v_i(z) = \phi'_{\mu}(\mu, x_i, F_i(x)) : i \in N\},$$
$$D_1(z) := \mathrm{diag}\{a_1(z), a_2(z), \ldots, a_n(z)\},$$
$$D_2(z) := \mathrm{diag}\{b_1(z), b_2(z), \ldots, b_n(z)\},$$