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# Estimation of risk-neutral processes in single-factor jump-diffusion interest rate models

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ABSTRACT

The estimation of the market price of risk is an open question in the jump-diffusion term structure literature when a closed-form solution is not known. Furthermore, the estimation of the physical drift has a high risk of misspecification. In this paper, we obtain some results that relate the risk-neutral drift and the risk-neutral jump intensity of interest rates with the prices and yields of zero-coupon bonds. These results open a way to estimate the drift and jump intensity of the risk-neutral interest rates directly from data in the markets. These two functions are unobservable but their estimations provide an original procedure for solving the pricing problem. Moreover, this new approach avoids the estimation of the physical drift as well as the market prices of risk. An application to US Treasury Bill data is illustrated.



## 1. Introduction

In traditional jump-diffusion interest rate models the drift, volatility and market prices of risk are usually specified as simple parametric functions for pure simplicity and tractability. Moreover, most models combine well-known parametric diffusion models with different jump size distributions. For example, [1] combine the square root process of Cox et al. [2] with a constant jump size, Baz and Das [3] and Beliaeva et al. [4] add exponential jumps to an Ornstein–Uhlenbeck process for the short rate given by Vasiceck [5] and so on. However, there is neither evidence nor consensus about which model is the best for accurately explaining the behaviour of interest rates. Furthermore, the functions of the models are usually chosen to obtain a closed-form solution for the pricing problem. As a result, Bandi and Nguyen [6], Johannes [7] and Mancini and Reno [8] proposed nonparametric jump-diffusion models of the short rates that nest most jump-diffusion processes, but it is not possible to obtain a closed-form solution for the pricing problem. However, there are a lot of efficient numerical methods to provide accurate approximated solutions to the pricing problem.

In order to obtain the yield curves when a closed-form solution is not known, the Monte Carlo method is often used in the literature because of its simplicity and properties. When we use the Monte Carlo method, we have to estimate the drift and jump intensity of the risk-neutral interest rate. This risk-neutral stochastic process is not observable in the markets and, therefore, in the diffusion literature the risk-neutral drift is usually obtained by estimating the drift and volatility of the instantaneous interest rates and market price of risk, separately. However, this is not possible in the jump-diffusion literature. The usual absence of arbitrage argument can no longer be applied, as jump risk cannot be diversified away using traded bonds, Nawalkha et al. [9]. Therefore, it is not possible to estimate the market prices of risk unless a closed-form

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solution is known. In diffusion models there is another alternative which consists of estimating the risk-neutral drift directly form market data, as in [10].

The main goal of this paper is to show results to estimate the drift and the jump intensity of the risk-neutral interest rates directly from market data in a jump-diffusion model. These results open up a way to solving the problem of pricing interest rate derivatives efficiently and without arbitrary restrictions regarding the functions of interest rate models. In the nonparametric jump-diffusion literature, the estimation of the market prices of risk problem has not been solved yet, but their values are arbitrary set, as in [7]. However, an incorrect specification of the market prices of risk can have dramatic consequences for derivatives valuation: see [11,12] for more details. In our approach, the market prices of risk of Wiener and Poisson processes do not have to be estimated. Furthermore, we do not have to estimate the physical drift either, whose estimation has a high risk of misspecification.

This approach can be used for parametric as well as nonparametric models. In this paper, we use nonparametric methods such as kernel methods to avoid any arbitrary restriction in the whole model.

In order to show the finite sample properties of our approach, we make certain numerical experiments to obtain the yield curves in a test problem. Finally, we show the superiority of our approach for a nonparametric jump-diffusion term structure model with US interest rate data.

The rest of the article is organized as follows: Section 2 presents the jump-diffusion interest rate model to be studied. Section 3 shows a new approach for estimating the drift and jump intensity of the risk-neutral interest rates by means of the slope of the yield curve and zero-coupon bonds with numerical differentiation. We will call this new approach GNEJ: General Nonparametric Estimation for Jumps. Section 4 analyzes the finite sample performance of this approach using numerical differentiation and a nonparametric method. Section 5 examines empirically the behaviour of the GNEJ approach with US interest rates data. Conclusions are contained in Section 6.

## 2. The term structure model

In this section, we present a jump-diffusion term structure model with only one state variable. Although one-factor models have several shortcomings, they are still very attractive for practitioners and academics because they offer a unifying tool for the pricing of many interest rate derivatives.

Let $(\Omega, \mathcal{F}, \mathcal{P})$ be a probability space equipped with a filtration $\mathcal{F}$ satisfying the usual conditions. The price of an interest rate security is driven by the instantaneous interest rate, which follows a mixed jump-diffusion stochastic process of the type:

$$dr(t) = \mu(r(t))dt + \sigma(r(t))dW(t) + J(r(t), Y(t))dN(t), \tag{1}$$

where $\mu(r)$ is the drift, $\sigma(r)$ is the volatility, $J(r, Y)$ is a function of the instantaneous interest rate and the magnitude of the jump $Y$, which is a random variable with probability distribution $\Pi$, $W$ is the Wiener process and $N$ represents a Poisson process with intensity $\lambda(r)$. Moreover, $dW$ is assumed to be independent of $dN$, which means that the diffusion component and the jump component of the short-term interest rates are independent of each other. We assume that jump magnitude and jump arrival times are uncorrelated with the diffusion part of the process. We assume that $\mu, \sigma, \lambda, J$ satisfy enough technical regularity conditions: see [13]. Under the above assumptions, the price at time $t$ of a zero-coupon bond maturing at time $T$, with $t \leq T$, can be expressed as $P(t, r; T)$. This bond is assumed to have a maturity value of one unit, i.e.:

$$P(T, r; T) = 1. \tag{2}$$

We also assume that there exists a new measure $\mathcal{Q}$, equivalent to $\mathcal{P}$, such that the price of a zero-coupon bond is

$$P(t, r; T) = E^{\mathcal{Q}}\left[\exp\left(-\int_t^T r(u)du\right) |\mathcal{F}(t)\right], \tag{3}$$

where $E^{\mathcal{Q}}$ denotes the conditional expectation under measure $\mathcal{Q}$ which is known as risk-neutral probability measure. Under $\mathcal{Q}$, the short-rate $r$ follows the process:

$$dr = \left(\mu(r) - \sigma(r)\theta^W(r) + \lambda^{\mathcal{Q}}(r)E_Y[J(r, Y)]\right) dt + \sigma(r)dW^{\mathcal{Q}}(t) + J(r, Y)d\tilde{N}^{\mathcal{Q}}(t),$$

where $W^{\mathcal{Q}}$ is the Wiener process under $\mathcal{Q}$, $\theta^W$ is the market price of risk of the Wiener process, $\tilde{N}^{\mathcal{Q}}$ represents the compensated Poisson process, under $\mathcal{Q}$ measure, with intensity $\lambda^{\mathcal{Q}}(r) = \lambda(r)\theta^N(r)$. For simplicity and tractability, we assume that the distribution of the jump size under $\mathcal{Q}$ is known and equal to distribution under $\mathcal{P}$, as usual in the literature, see [14,9,12]. That is, we assume that all risk premium related to jump risk is artificially absorbed by the change in the intensity of jump from $\lambda$ under the physical measure to $\lambda^{\mathcal{Q}}$ under the risk-neutral measure.

In the absence of the jump component, it is possible to derive the term structure equation by constructing a riskless portfolio of two bonds of different maturities and imposing the absence of the arbitrage condition. However, in the presence of a jump component, the absence of arbitrage argument can no longer be applied, as jump risk cannot be diversified away using traded bonds, Nawalkha et al. [9]. Therefore, the valuation of fixed income securities requires transition from actual to the equivalent martingale measure. In general, this task can be accomplished by specifying a stochastic discount factor for