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An improved hybrid ant-local search algorithm for the partition graph coloring problem

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ABSTRACT

In this paper we propose a hybrid Ant Colony Optimization (ACO) algorithm for the Partition Graph Coloring Problem (PGCP). Given an undirected graph $G = (V, E)$, whose nodes are partitioned into a given number of the sets, the goal of the PGCP is to find a subset $V^* \subset V$ that contains exactly one node from each cluster and a coloring for V^* so that in the graph induced by V^*, two adjacent nodes have different colors and the total number of used colors is minimal. Our hybrid algorithm is obtained by executing a local search procedure after every ACO iteration. The performance of our algorithm is evaluated on a set of instances commonly used as benchmark and the computational results show that compared to state-of-the-art algorithms, our improved hybrid ACO algorithm achieves solid results in very short run-times.

1. Introduction

In this paper we consider the partition graph coloring problem, denoted by PGCP. Given an undirected graph $G = (V, E)$ and a partition of its nodes into p node sets $V_1, \ldots, V_p$, called clusters, the *partition graph coloring problem* consists in finding a subset $V^* \subseteq V$ containing exactly one node from each cluster V_i, $i \in \{1, \ldots, p\}$ and such that the chromatic number of the graph induced in G by V^* is minimum.

The considered problem belongs to a class of combinatorial optimization problems commonly referred to as generalized network design problems (GNDPs) or generalized combinatorial optimization problem (GCOPs). This class of problems is obtained in a natural way, generalizing many combinatorial optimization problems by considering a related problem on a clustered graph (i.e. a graph whose nodes are partitioned into a given number of clusters), where the original problem's feasibility constraints are expressed in terms of the clusters, i.e., node sets instead of individual nodes. For more information concerning to this class of optimization problems we refer to [1,2].

In the last period several generalized combinatorial optimization problems have been studied such as the generalized minimum spanning tree problem [3,4], the generalized traveling salesman problem [5], the generalized vehicle routing problem [6,7], the partition graph coloring problem [8], and the generalized fixed-charge network design problem [9]. All such problems belong to the class of $\mathcal{NP}$-complete problems, they are harder to solve in practice than their original counterparts and recently a lot of research is emphasized on them especially due to their interesting properties and important

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real-world applications in telecommunication, network design, scheduling problems, resource allocation, transportation problems, software engineering, etc.

The PGCP was introduced by Li and Simha and it was motivated by considering the joint problem of routing and assignment in wavelength division multiplexing optical networks [8]. The same authors proved that the problem is $\mathcal{NP}$-complete. Demange et al. [10,11] considered this type of graph coloring problem in the framework of generalized network design problems and named it selective graph coloring problem. They investigated as well some special classes of graphs including split graphs, bipartite graphs and q-partite graphs and settled the complexity status of the PGCP in these particular classes.

Due to its practical applications and its complexity, the partition graph coloring problem has generated an important interest, being proposed exact and heuristic algorithms: Li and Simha [8] designed two groups of heuristic algorithms: one-step algorithms including *onestep Largest-First*, *onestep Smallest-Last* and *onestep Color-Degree* and two-steps algorithms including *twosteps Largest-First*, *twosteps Smallest-Last* and *twosteps Color-Degree*, Frota et al. [12] described a branch-and-cut algorithm for PGCP, Hoshino et al. [13] proposed an integer programming model and a branch-and-price algorithm to solve it, Noronha and Ribeiro [14] described a Tabu Search algorithm. Recently, Pop et al. [15] proposed a memetic algorithm (MA) which uses two different solution representations for the genetic operators and for the local search procedure.

The aim of this paper is to present an efficient ant colony algorithm for solving the PGCP. In addition, we proposed an improved hybrid ACO algorithm, which was obtained by combining a traditional pure ACO algorithm with a local search procedure. We compare the performance of our proposed hybrid ACO and pure ACO algorithms with other metaheuristic methods on a set of instances commonly used as benchmark.

The remainder of the paper is organized as follows. Section 2 provides some definitions and notations used throughout the paper and formally state the partition graph coloring problem. Section 3 describes the ACO framework. Section 4 introduces the local search procedure and the hybrid ACO algorithm. Section 5 presents and analyzes the results of the computational experiments and finally, the last section concludes the paper.

2. Definition of the partition graph coloring problem

We start this section with some basic definitions concerning graph coloring. For more details we refer for example to [16].

Let $G = (V, E)$ be an undirected graph and let $V' \subseteq V$ then the *graph induced by* V' is obtained from the graph G by deleting the nodes of $V \setminus V'$ and the all the edges incident to at least one node from the set $V \setminus V'$.

A *vertex k-coloring* of the graph G is a mapping $c : V \to \{1, \ldots, k\}$ with the property that $c(u) \neq c(v)$ for all the edges $(u, v) \in E$. The number $c(u)$ or $c(v)$ is called the *color* of u or v. A graph that can be assigned a k-coloring is *k-colorable*.

The *vertex-coloring problem* consists in finding a vertex-coloring of G with minimum k. The smallest number of colors needed to color a graph G is called *chromatic number*.

Formally, the partition graph coloring problem is defined on an undirected graph $G = (V, E)$ with the set of nodes V and the set of edges E. The set of nodes is partitioned into p mutually exclusive nonempty subsets, called clusters, $V_1, \ldots, V_p$ with $V_1 \cup \cdots \cup V_p = V$ and $V_i \cap V_j = \emptyset$ for all $i, j \in \{1, \ldots, p\}$ and $i \neq j$. The PGCP consists of finding a set $V^* \subset V$ such that:

1. $|V^* \cap V_i| = 1$, i.e., V^* contains exactly one node from each cluster V_i for all $i \in \{1, \ldots, p\}$,
2. the graph induced by V^* is k-colorable where k is minimal.

The PGCP reduces to the classical graph coloring problem when all the clusters are singletons.

An illustration of the PGCP, a feasible solution with three colors and an optimal solution with two colors, is shown in Fig. 1.

In this example the graph $G = (V, E)$ has 8 nodes partitioned into 4 clusters. A feasible solution for the PGCP making use of three colors is represented. The optimal solution makes use of two colors: the first is used to color the nodes 3 and 8 and the second for the nodes 5 and 7.

3. The ACO algorithm

Ant Colony Optimization is one of the most successful metaheuristic methods. The main idea comes from collective intelligence of real ants when they look for a food. The problem is solved collectively by the whole colony. This ability is explained by the fact that ants communicate in an indirect way by laying trails of pheromone on the ground. If the pheromone trail within a particular direction is higher, the probability of choosing this direction is higher.

The ACO algorithm was proposed by Dorigo et al. [17,18]. It uses a colony of artificial ants that behave as cooperative agents in a mathematical space where they are allowed to search and reinforce pathways (solutions) in order to find the optimal ones. The problem is represented by a graph and where the ants can walk for constructing solutions. Solutions are therefore represented by paths in this graph. After the initialization of the pheromone trails, the ants construct feasible solutions, starting from random nodes, and then the pheromone trails are updated. At each step the ants compute a set of feasible moves and select the best one (according to some probabilistic rules) to continue the rest of the tour. The transition