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Scalable parallel implementation of shooting method for large-scale dynamical systems. Application to bridge components

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ABSTRACT

The mathematical models applied to real-life engineering structures result into large-scale dynamical systems. The efficient computation of their dynamical characteristics requires the usage of advanced numerical methods with parallel algorithms. The shooting method, in combination with a continuation method, presents a powerful tool for analyzing and investigating the dynamical characteristics of such systems. The efficiency and scalability of the shooting method are analyzed in the current paper for linear and nonlinear equations. Its applicability to large-scale systems is demonstrated by structural component of bridge discretized by three-dimensional finite elements.

1. Introduction

The knowledge of dynamical behavior of elastic structures is essential for their design, analysis and maintenance. Different approaches and mathematical models have been developed for that purpose in the last decades. Enormous attention was paid to nonlinear dynamical systems because of their complex behavior and the necessity of efficient numerical methods for solving the equations.

The main characteristics of linear and nonlinear dynamical systems are summarized briefly in this paragraph. The principle of superposition holds for linear systems, the time response due to harmonic excitation is also harmonic, with frequency equal to the excitation frequency, and the steady-state solution is unique and independent on the initial conditions. For nonlinear systems, the principle of superposition does not hold, the response of nonlinear systems can be periodic, quasi periodic or chaotic and there might exists more than one steady-state solution, which depends on the initial conditions. In addition, the nonlinear systems can have bifurcation points which can change the solution significantly. The nonlinear systems can exhibit complex motions which linear systems cannot. More details about the characteristics of nonlinear dynamical systems can be found in [1,2].

The linear systems are much easier to solve, but they have limited application. They are appropriate for small displacements. In many cases linearity is a rude approximation of the real problems. The necessity of using nonlinear models comes from the requirement of engineers to use more accurate and better physical models. In most cases, it is impossible to obtain analytical solution of nonlinear equations, thus one has to use appropriate numerical methods [3].

Apart from the tendency of developing more accurate physical models by considering nonlinear terms in the equation of motion, another necessity for the engineers is to obtain more precise solutions of nonlinear partial differential equations

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(PDE) on complex domains. A standard technique for transforming the PDE into a system of ordinary differential equations (ODE) is the finite element method [4]. Accuracy is achieved by using a fine mesh of finite elements or by increasing the order of the polynomials, used as shape functions. Both cases result into large systems of ODE. Analyzing the dynamical behavior of large-scale systems of nonlinear ordinary differential equations is a challenging and ambitious task.

The variation of the solution with the excitation frequency (or frequency of vibration) is of primary interest for the analysis of the dynamical behavior of elastic structures [5]. The concepts of nonlinear normal modes (NNM) and non-linear frequency-response function (FRF) have become standard tools for such analyses [6]. Each point from the frequency–amplitude (or frequency–energy) domain presents a periodic solution. The existence of quasi periodic solutions is determined by appearance of secondary Hopf bifurcation points [1,3]. A transition to chaotic response is also possible. The existence of cyclic-fold, subcritical period-doubling or subcritical Hopf bifurcation points is a preliminary requirement for appearance of chaos in the system [1].

There exist several methods for obtaining periodic responses of dynamical systems. The brute-force approach is the simplest one which uses a time integration scheme [7]. Another common approach is the harmonic balance method (HBM) where the solution is expressed in Fourier series and balance of harmonics is applied [8,9]. Periodic solutions can also be obtained by collocation method [10]. Shooting method computes iteratively the initial conditions which lead to periodic response [1]. Advantages of each of these methods are discussed in [11]. In this work, the shooting method is preferred for the analysis of large-scale dynamical systems because it is very suitable for implementation on distributed memory machines for parallel computing.

The purpose of the paper is to present an efficient parallel realization of the shooting method for obtaining periodic responses of large-scale dynamical systems. The Bernoulli–Euler beam equation of motion is used as a model equation for studying scalability of the shooting method for linear and nonlinear systems. The potential of the proposed method to real-life applications is demonstrated on bridge structural component using three-dimensional finite elements and the linear equation of motion.

Even though the proposed parallel implementation of the method is presented to forced vibrations, it is not limited to computing the nonlinear frequency-response function. It can also be applied to free vibration problems and for computing the nonlinear normal modes. In this case, predictor and corrector for the period of vibration should be defined, but the suggested parallel implementation can remain the same.

2. Parallel implementation of shooting method

The shooting method is applied to systems of second order ODE. This formulation is preferred here, instead of the common formulation for systems of first order ODE, because of two reasons. First, the CPU time is significantly reduced, because the size of the independent systems (these systems arise from the shooting method and are shown in 2.1) remains equal to the size of the initial system, while if transformation into a system of first order ODE is performed, the size of the independent systems becomes double. Second, additional computations which arise from solving the system of first order ODE are not performed. These computations are related with the mass matrix. The computation of the inverse of the mass matrix can lead to loss of accuracy, it is time consuming and the resulting matrix is not sparse, hence it requires a lot of memory. Thus, such computation should be avoided and the mass matrix should remain on the left hand side of the system. On the other side, by keeping the mass matrix on the left hand side of the system, one still has to do additional computations for obtaining a numerical solution of the system. Thus, the application of the shooting method to large-scale systems of second order ODE is much more efficient when it is applied directly to the original system.

A parallel implementation of the shooting method on multiple processors is proposed in this section. The sparse and dense matrices, which take part in the method, are pointed out and the distribution of the computations and memory among the processes is emphasized. Discussion about the scalability of the method to linear and nonlinear systems is presented.

2.1. Shooting method for systems of second order ODE

The nonlinear equation of motion of elastic structure, after space discretization, has the following form:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}(\mathbf{q}(t))\,\mathbf{q}(t) = \mathbf{f}(t), \tag{1}$$

where $\mathbf{q}(t)$ is the vector of generalized coordinates, $\mathbf{M}$ represents the mass matrix, $\mathbf{C}$—the damping matrix, $\mathbf{K}(\mathbf{q}(t))$—the stiffness matrix that depends on vector $\mathbf{q}(t)$, $\mathbf{f}(t)$ is the generalized vector of external forces. A harmonic excitation is considered, thus the vector of external forces is written as $\mathbf{f}(t) = \mathbf{a}\cos(\omega t)$, where ω is the frequency of excitation. The total number of degrees of freedom (DOF) of the equation of motion (1) is denoted by N. The equation of motion has initial conditions denoted by:

$$\begin{aligned} \mathbf{q}(0) &= \mathbf{q}_0, \\ \dot{\mathbf{q}}(0) &= \dot{\mathbf{q}}_0. \end{aligned} \tag{2}$$

The shooting method consists of iterative computation of the initial conditions $\mathbf{q}_0$ and $\dot{\mathbf{q}}_0$ which perform a periodic motion without transient response.

Let the period of vibration be denoted by T. For linear systems $T = 2\pi/\omega$ and for nonlinear systems the period of vibration is an integer multiplied by $2\pi/\omega$. The equation of motion (1) is integrated in time domain, for one period of vibration. In