Journal of Computational and Applied Mathematics 288 (2015) 70–80

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

New method for computing the upper bound of optimal value in interval quadratic program

Wei Li [a], Mengxue Xia [a], Haohao Li [b,*]

[a] Institute of Operational Research & Cybernetics, Hangzhou Dianzi University, Hangzhou, 310018, China

[b] School of Mathematics and Statistics, Zhejiang University of Finance and Economics, Hangzhou, 310018, China

ARTICLE INFO

Article history:
Received 26 January 2015
Received in revised form 29 March 2015

MSC:
15A06
65G40

Keywords:
Quadratic programming
Interval systems
Optimal value range
Duality gap

ABSTRACT

We consider the interval quadratic programming problems. The aim of this paper is to present a new method to compute the upper bound of the optimal values, under weaker conditions. Moreover, we discuss the relations between the new method and previous results. The features of the proposed methods are illustrated by some examples.

1. Introduction

In many real world applications, system parameters or model coefficients are not always known exactly and may be bounded between lower and upper bounds due to a variety of uncertainties [1–3]. Over the past decades, interval mathematical programming methods were developed to tackle such uncertainties [4–13]. Many papers studied the problem of computing the range of optimal values of interval linear programming problems, see e.g., [7,10,14–17] among others. Some authors studied the problem of computing the range of optimal values of interval quadratic programs (IvQP). It is known that finding the lower bound of the optimal values in IvQP is polynomially solvable, whereas finding the upper bound of the optimal values is a computationally hard problem when the constraints include interval linear equalities. While to determine the upper bound of the optimal values, the existing methods have to consider the dual of the primal problem, and the condition that the duality gap is zero should be specified [18–20].

We study IvQP and our aim is to establish a new method to compute the upper bound of optimal values, which is an analogue of the results in interval linear program [7,15,16]. In this method, only primal program is taken into consideration. The dual problem is not required and thus the condition that the duality gap is zero is also removed.

2. Preliminaries

Following notations from [15], an interval matrix is defined as

$$\mathbf{A} = [\underline{A}, \overline{A}] = \{A \in \mathbb{R}^{m\times n} : \underline{A} \le A \le \overline{A}\},$$

* Corresponding author.
E-mail address: hhli@zufe.edu.cn (H. Li).

http://dx.doi.org/10.1016/j.cam.2015.03.055

where $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, $\underline{A} \le \overline{A}$, and "$\le$" is understood componentwise. By

$$A_c = \frac{1}{2}(\underline{A} + \overline{A}), \qquad A_\Delta = \frac{1}{2}(\overline{A} - \underline{A}),$$

we denote the center and the radius of A, respectively. Then $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$. An interval vector $\mathbf{b} = [\underline{b}, \overline{b}] = \{b \in \mathbb{R}^m : \underline{b} \le b \le \overline{b}\}$ is understood as one-column interval matrix.

Let $\{\pm 1\}^m$ be the set of all $\{-1, 1\}m$-dimensional vectors, i.e.

$$\{\pm 1\}^m = \{y \in \mathbb{R}^m \mid | y |= e\},$$

where $e = (1, \dots, 1)^T$ is the m-dimensional vector of all 1's and the absolute value of a matrix $A = (a_{ij})$ is defined by $|A| = (|a_{ij}|)$. For a given $y \in \{\pm 1\}^m$, let

$$T_y = diag(y_1, \dots, y_m)$$

denote the corresponding diagonal matrix. For each $x \in \mathbb{R}^n$, we define its sign vector sgn x by

$$(\text{sgn}\, x)_i = \begin{cases} 1 & \text{if } x_i \ge 0, \\ -1 & \text{if } x_i < 0, \end{cases}$$

where $i = 1, \dots, n$. Then we have $|x| = T_z x$, where $z = \text{sgn}\, x \in \{\pm 1\}^n$.

Given an interval matrix $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$, for each $y \in \{\pm 1\}^m$ and $z \in \{\pm 1\}^n$, we define matrices

$$A_{yz} = A_c - T_y A_\Delta T_z.$$

Similarly, for an interval vector $\mathbf{b} = [b_c - b_\Delta, b_c + b_\Delta]$ and for each $y \in \{\pm 1\}^m$, we define vectors

$$b_y = b_c + T_y b_\Delta.$$

Let $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{k\times n}$, $b \in \mathbb{R}^m$, $c \in \mathbb{R}^n$, $d \in \mathbb{R}^k$ and $Q \in \mathbb{R}^{n\times n}$ be given, consider the quadratic programming problem

$$\min \frac{1}{2}x^T Qx + c^T x \quad \text{subject to } Ax \le b, \quad Bx = d, x \ge 0,$$

where Q is positive semidefinite. Briefly, we rewrite the problem as

$$\text{Min} \left\{ \frac{1}{2}x^T Qx + c^T x | Ax \le b, Bx = d, x \ge 0 \right\}. \tag{1}$$

The Dorn dual problem [21,22] of the quadratic program (1) is

$$\text{Max} \left\{ -\frac{1}{2}u^T Qu - b^T v - d^T w | Qu + A^T v + B^T w + c \ge 0, v \ge 0 \right\}. \tag{2}$$

Let

$$f(A, B, b, c, d, Q) = \inf \left\{ \frac{1}{2}x^T Qx + c^T x | Ax \le b, Bx = d, x \ge 0 \right\}$$

and

$$g(A, B, b, c, d, Q) = \sup \left\{ -\frac{1}{2}u^T Qu - b^T v - d^T w | Qu + A^T v + B^T w + c \ge 0, v \ge 0 \right\}$$

denote the optimal value of (1) and (2), respectively.

Clearly, the following result of weak duality holds.

Theorem 2.1 (*Weak Duality*). *We have*

$$f(A, B, b, c, d, Q) \ge g(A, B, b, c, d, Q).$$

The following result of strong duality is from [21].

Theorem 2.2 (*[21]*). (i) *If $x = x_0$ is an optimal solution to problem (1) then an optimal solution $(u, v, w)^T = (u_0, v_0, w_0)^T$ exists to problem (2).* (ii) *Conversely, if an optimal solution $(u, v, w)^T = (u_0, v_0, w_0)^T$ to problem (2) exists then an optimal solution $x = x_0$ to problem (1) also exists. In either case,*

$$f(A, B, b, c, d, Q) = g(A, B, b, c, d, Q).$$