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A generalized project metric algorithm for mathematical programs with equilibrium constraints[☆]

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ABSTRACT

This paper discusses a kind of mathematical programs with equilibrium constraints (MPEC for short). By using a complementarity function and a kind of disturbed technique, the original (MPEC) problem is transformed into a nonlinear equality and inequality constrained optimization problem. Then, we combine a generalized gradient projection matrix with penalty function technique to given a generalized project metric algorithm with arbitrary initial point for the (MPEC) problems. In order to avoid Mataros effect, a high-order revised direction is obtained by an explicit formula. Under some relative weaker conditions, the proposed method is proved to possess global convergence and superlinear convergence.

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1. Introduction

We consider the following (MPEC) in which the constraints are defined by a nonlinear complementarity problem as follows:

$$\begin{array}{ll} \min & f(x,y) \\ \text{s.t.} & g(x,y) \le 0, \\ & h(x,y) = 0, \\ & 0 \le F(x,y) \perp y \ge 0, \end{array} \tag{1.1}$$

where $f : R^{n+m} \to R$, $g = (g_1, \ldots, g_l)^T : R^{n+m} \to R^l$, $h = (h_1, \ldots, h_p)^T : R^{n+m} \to R^p$ and $F = (F_1, \ldots, F_m)^T : R^{n+m} \to R^m$ are continuously differentiable, and $a \perp b$ denotes orthogonality of any vectors $a, b \in R^n$, i.e., $a^\top b = 0$. Such problems play an important role in many fields such as the design of transportation networks, economic equilibrium and engineering design, see [1,2].

Due to the existence of the complementarity constraints, it is a very difficult research to solve the (MPEC). There have been many scholars to study the (MPEC). Outrata et al. [3] pointed out that the Mangasarian–Fromovitz constraint qualification (MFCQ for short) does not hold at any feasible point of this (MPEC). So the theory for nonlinear programming cannot be directly applied to the problem (1.1), hence standard methods are not guaranteed to solve such problems. There have been many achievements to solve the (MPEC), such as Fukushima, et al. [4] proposed a class of SQP algorithms for the solution of a kind of mathematical program with linear complementarity constraints (MPLCC for short). Their algorithms possess global

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convergence under a strict complementarity condition on the lower-level constraints (or call it lower-level nondegeneracy). Recently, Fukushima and Tseng [5] proposed an ε-active set to approach the discussed (MPLCC). Under a uniform (LICQ) on the ε-feasible set, the proposed algorithm is proved to be globally convergent. In 2000, Jiang and Ralph [6] extended the method of [4] to the (MPEC) (both the upper-level and lower-level constraints are nonlinear). However, they only possess global convergence. In addition to, using complementarity function with a perturbed parameter $\mu(\mu > 0)$, a kind of smoothing method was proposed in [6–10], by solving a sequence smoothing optimization to approximate successfully the solution of the (MPEC). Under the MPEC-LICQ and asymptotically weak nondegeneracy, an accumulated point of the perturbed problem is a stationary point of the original (MPEC) problem. So from this point, it is important for us to obtain the stationary point of the original (MPEC) problem while constructing an algorithm to solve it.

The gradient projection method (GPM for short), which was first developed by Rosen in 1960s, is one of the early important methods of feasible directions for solving the nonlinear inequality constrained optimization problem. Thereafter, many authors (see, e.g., [11–18]) further researched and improved the GPM, especially in the case of nonlinear constraints. In the recent two decades, a new kind of projection type method collectively called generalized gradient projection methods (GGPMs) have been proposed, see, e.g., [13,16–18].

Motivated by the ideas of [13,16] in this paper, we present here a generalized project metric algorithm with arbitrary initial point for Mathematical programs with equilibrium constraints. By means of the perturbed technique and a generalized complementarity function $\varphi(a, b, \mu) = a+b-\sqrt{a^2+b^2+2\mu},\ (a, b) \in R^2$, we transform equivalently problem (1.1) into a nonlinear equality and inequality constrained optimization problem. Then, we combine a conjugate projection matrix with penalty function technique to given a conjugate projection gradient method for the (MPEC) problem. Compared with the existing methods for MPEC, the master search direction and a correction search direction for avoiding the Maratos effect are generated by an explicit formula. Under suitable conditions, the proposed algorithm is proved to possess global convergence and superlinear convergence rate. Our numerical results show the efficiency of the proposed algorithm.

2. Preliminaries, equivalent reformulations and algorithm

The problem (1.1) is equivalent to the following reformulations:

$$\begin{array}{ll} \min & f(x,y) \\ \text{s.t.} & g(x,y) \le 0, \\ & h(x,y) = 0, \\ & w = F(x,y), \\ & w^T y = 0. \end{array} \tag{2.1}$$

The symbols we use in this paper are standard. For convenience, we list some of them as follows:

$$\begin{array}{lllll} z = (x, y, w), & s = (x, y), & t = (y, w), & X_0 = \{z : g(x,y) \le 0,\ h(x,y)0,\ w = F(x,y),\ w^T y = 0\}, & \\ z^k = (x^k,\ y^k,\ w^k), & s^k = (x^k, y^k), & t^k = (y^k, w^k), & & \\ dz = (dx, dy, dw), & ds = (dx, dy), & dt = (dy, dw), & & \\ dz^k = (dx^k,\ dy^k, dw^k), & ds^k = (dx^k, dy^k), & dt^k = (dy^k, dw^k), & I = \{1, \dots, l\}, & \\ L = \{l+1, \dots, l+p+2m\}, & L_0 = L \cup I. & & & \end{array}$$

Throughout this paper, we suppose that the following assumption holds:

H 2.1. The functions $f(x, y)$, $g(x, y)$, $h(x, y)$ and $F(x, y)$ are all twice continuously differentiable.

Proposition 2.1. (i) *A feasible point $z^* = (x^*, y^*, z^*) \in X_0$ is said to be a stationary point of the MPEC (1.1) if*

$$\nabla f(s^*)^T ds \ge 0, \quad \forall ds \in T(z^*, X_0),$$

where $T(z^*, X_0)$ denotes the tangent cone of X_0 at point z^* [2].

(ii) *Suppose that $z^* \in X_0$ satisfies the so-called nondegeneracy condition:*

$$(y_i^*, w_i^*) \ne (0, 0), \quad i = 1, 2, \dots, m, \tag{2.2}$$

then z^* is a KKT stationary point of the problem (2.1) if and only if there exist KKT multipliers $(\lambda^*, \eta^*, u^*, \upsilon^*) \in R^l \times R^p \times R^m \times R^m$ such that

$$\begin{aligned} &\begin{pmatrix} \nabla f(s^*) \\ 0 \end{pmatrix} + \begin{pmatrix} \nabla g(s^*) \\ 0 \end{pmatrix} \lambda^* + \begin{pmatrix} \nabla h(s^*) \\ 0 \end{pmatrix} \eta^* + \begin{pmatrix} \nabla F(s^*) \\ -E_m \end{pmatrix} u^* + \begin{pmatrix} 0 \\ W^* \\ Y^* \end{pmatrix} \upsilon^* = 0, \\ &0 \le -g(s^*) \perp \lambda^* \ge 0, \end{aligned} \tag{2.3}$$

where W^* and Y^* are diagonal matrices with diagonal entries w_j^* and $y_j^*,\ j = 1 \sim m$, respectively.

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