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A characterization of multivariate normal stable Tweedie models and their associated polynomials

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ABSTRACT

Multivariate normal stable Tweedie models are recently introduced as an extension to normal gamma and normal inverse Gaussian models. The aim of this paper is to characterize these models through their variance functions. Then, according to the power variance parameter values, the nature of polynomials associated with these models is deduced.
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Contents

1. Introduction

An important problem in statistical analysis is how to choose an adequate family of distributions or statistical model to describe the study. For this purpose, the characterization theorems can be useful because, under general reasonable

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suppositions related to the nature of the experiment, they allow us to reduce the possible set of distributions that can be used. One of these reasonable assumptions is that the normal stable Tweedie (NST) models [1] present particular forms of variance functions based on the first component of mean vector and a probability measure which is not easy to handle. So, a characterization by variance functions or by associated polynomials is required for the analysis related to this model. Recall that variance function plays a significant role in the classification of natural exponential families (NEFs). Thus, the NEFs can be characterized by variance functions obtained by successive differentiations of the Laplace transform of a positive measure. Also, the variance functions are convenient to identify a family that is, for example, a Laplace transform to identify a probability distribution.

Several methods of classification of NEFs are proposed by some authors, among which may be mentioned those of Morris [2] which classified all the NEFs with quadratic variance functions in the one dimensional case. This classification comprises some of the most common distributions like normal, Poisson, gamma, binomial, and negative binomial distributions. Different characterizations of the Morris class involving orthogonal polynomials are due to Feinsilver [3], Meixner [4] and Shanbhag [5]. An extension of these characterizations of NEFs in univariate and multivariate are also given these last years by some authors. In univariate case for example, Letac and Mora [6] classified the real NEFs with cubic variance functions and Hassairi and Zarai [7] characterized them by a property of 2-orthogonality. Kokonendji [8,9] characterized by d-pseudo-orthogonality and by d-orthogonality of the Sheffer systems of the NEFs with polynomials variance functions of degree $2d-1$ which are particular cases of the univariate stable Tweedie [10] class.

In the framework of multivariate NEFs, we first refer to Letac [11] which characterizes the Poisson–Gaussian families by variance functions; see also [12] for the characterization of these families by determinant of covariance matrix, called generalized variance function. More generally Casalis [13] classified all the $2d+4$ simple quadratic NEFs on $\mathbb{R}^d$, which are the $d+1$ types of Poisson–Gaussian families, the $d+1$ types of negative multinomial-gamma-Gaussian families (see also [14] for generalized variance function of the gamma-Gaussian type), the multinomial family and the hyperbolic family. See also [15] for characterization of simple quadratic NEFs with a reverse martingale property. Also, Bar-Lev et al. [16] characterized in six types the irreducible diagonal NEFs in $\mathbb{R}^d$ which are: normal, Poisson, multinomial, negative multinomial, gamma and hybrid. About characterizations via polynomials we can cite the works of: Pommeret [17,18] for the characterization of simple quadratic (resp. quadratic) NEFs by the orthogonal (resp. pseudo-orthogonal) polynomials or Sheffer systems; Hassairi and Zarai [19] also Kokonendji and Zarai [20] for the transorthogonality or 2-pseudo-orthogonality of simple cubic multivariate NEFs; and, we finally mention the work of Kokonendji and Pommeret [21] on the characterization of multivariate NEFs with polynomial variance functions.

For an accurate presentation of this work, let us introduce some notations. Let $k \in \{2, 3, \dots\}$, we denote by $(\mathbf{e}_i)_{i=1,\dots,k}$ an orthonormal basis of $\mathbb{R}^k$ and by $\mathbf{I}_k = \mathbf{Diag}_k(1, \dots, 1)$ the $k \times k$ unit matrix. For two vectors $\mathbf{a} = (a_1, \dots, a_k)^\top \in \mathbb{R}^k$ and $\mathbf{b} = (b_1, \dots, b_k)^\top \in \mathbb{R}^k$, we use the notations $\langle \mathbf{a}, \mathbf{b} \rangle = \mathbf{a}^\top \mathbf{b}$ and $\mathbf{a} \otimes \mathbf{b} = \mathbf{ab}^\top$ to denote the scalar $\sum_{j=1}^k a_j b_j$ and the $k \times k$ matrix $\left(a_i b_j\right)_{i,j=1,\dots,k}$ respectively, and finally $\mathcal{S}\left(\mathbb{R}^k\right)$ the set of symmetric matrices on $\mathbb{R}^k$. Recall that, given a positive Radon measure $\boldsymbol{\mu}$ on $\mathbb{R}^k$, we will use the Laplace transform $L_{\boldsymbol{\mu}}$ and the cumulant function $\mathbf{K}_{\boldsymbol{\mu}}$ of $\boldsymbol{\mu}$ defined, respectively, by

$$L_{\boldsymbol{\mu}} : \mathbb{R}^k \to (0, \infty), \quad \boldsymbol{\theta} \mapsto L_{\boldsymbol{\mu}}(\boldsymbol{\theta}) := \int_{\mathbb{R}^k} \exp(\langle \boldsymbol{\theta}, \mathbf{x} \rangle) \boldsymbol{\mu}(\mathbf{dx})$$

and $\mathbf{K}_{\boldsymbol{\mu}}(\boldsymbol{\theta}) := \log L_{\boldsymbol{\mu}}(\boldsymbol{\theta})$ on the non-empty interior $\Theta(\boldsymbol{\mu})$ of the domain $\{\boldsymbol{\theta} \in \mathbb{R}^k; L_{\boldsymbol{\mu}}(\boldsymbol{\theta}) < \infty\}$. We then denote by $\mathcal{M}(\mathbb{R}^k)$ the set of σ-finite positive measures $\boldsymbol{\mu}$ not concentrated on an affine subspace of $\mathbb{R}^k$. Thus, for $\boldsymbol{\mu} \in \mathcal{M}(\mathbb{R}^k)$, the set of probability measures

$$\mathbf{F} = \mathbf{F}(\boldsymbol{\mu}) = \{\mathbf{P}(\boldsymbol{\theta}, \boldsymbol{\mu})(d\mathbf{x}) = \exp[\langle \boldsymbol{\theta}, \mathbf{x} \rangle - \mathbf{K}_{\boldsymbol{\mu}}(\boldsymbol{\theta})] \boldsymbol{\mu}(d\mathbf{x}); \;\; \boldsymbol{\theta} \in \Theta(\boldsymbol{\mu})\}$$

is called the NEF generated by $\boldsymbol{\mu}$. The measure $\boldsymbol{\mu}$ is called a basis of $\mathbf{F}$. For $\boldsymbol{\mu} \in \mathcal{M}(\mathbb{R}^k)$, $\mathbf{K}_{\boldsymbol{\mu}}$ is strictly convex and real analytic on $\Theta(\boldsymbol{\mu})$, and for all $\boldsymbol{\theta} \in \Theta(\boldsymbol{\mu})$ one has

$$\mathbf{K}'_{\boldsymbol{\mu}}(\boldsymbol{\theta}) = \left(\frac{\partial \mathbf{K}_{\boldsymbol{\mu}}(\boldsymbol{\theta})}{\partial \theta_j} \right)_{j=1,\dots,k} = \int_{\mathbb{R}^k} \mathbf{x} \mathbf{P}(\boldsymbol{\theta}, \boldsymbol{\mu}) d\mathbf{x} =: \mathbf{m}(\boldsymbol{\theta})$$

and then

$$\mathbf{K}''_{\boldsymbol{\mu}}(\boldsymbol{\theta}) = \left(\frac{\partial^2 \mathbf{K}_{\boldsymbol{\mu}}(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \right)_{i,j=1,\dots,k} = \int_{\mathbb{R}^k} [\mathbf{x} - \mathbf{m}(\boldsymbol{\theta})] \otimes [\mathbf{x} - \mathbf{m}(\boldsymbol{\theta})] \mathbf{P}(\boldsymbol{\theta}, \boldsymbol{\mu}) d\mathbf{x} =: \mathbf{V}_{\mathbf{F}}\Big(\mathbf{m}(\boldsymbol{\theta})\Big).$$

Both functions $\mathbf{m}(\boldsymbol{\theta})$ and $\mathbf{V}_{\mathbf{F}}\Big(\mathbf{m}(\boldsymbol{\theta})\Big)$ are, respectively, the mean and the variance–covariance matrix of $\mathbf{F}$. The function $\mathbf{K}'_{\boldsymbol{\mu}} : \Theta(\boldsymbol{\mu}) \to \mathbf{K}'_{\boldsymbol{\mu}}(\Theta(\boldsymbol{\mu})) =: \mathbf{M}_{\mathbf{F}}$ defines a diffeomorphism (see [22]), where $\mathbf{M}_{\mathbf{F}}$ denotes the mean domain of $\mathbf{F}$. So, let $\psi_{\boldsymbol{\mu}} : \mathbf{M}_{\mathbf{F}} \to \Theta(\boldsymbol{\mu})$ be the inverse function of $\mathbf{K}'_{\boldsymbol{\mu}}$. For all $\mathbf{m} = (m_1, \dots, m_k)^\top \in \mathbf{M}_{\mathbf{F}}$ and setting $\mathbf{P}(\mathbf{m}, \mathbf{F}) = \mathbf{P}(\psi_{\boldsymbol{\mu}}(\mathbf{m}), \boldsymbol{\mu})$ the probability of $\mathbf{F}$ with mean $\mathbf{m}$, we have $\mathbf{F} = \{\mathbf{P}(\mathbf{m}, \mathbf{F}); \mathbf{m} \in \mathbf{M}_{\mathbf{F}}\}$. Then, the covariance matrix of $\mathbf{P}(\mathbf{m}, \mathbf{F})$ can be written as follows:

$$\mathbf{V}_{\mathbf{F}}(\mathbf{m}) = \mathbf{K}''_{\boldsymbol{\mu}}\left(\psi_{\boldsymbol{\mu}}(\mathbf{m})\right) = \left[\psi'_{\boldsymbol{\mu}}(\mathbf{m})\right]^{-1} \in \mathcal{S}\left(\mathbb{R}^k\right).$$

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