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A generalized multiscale finite element method for the Brinkman equation

Juan Galvis [a,*], Guanglian Li [b], Ke Shi [b]

[a] *Departamento de Matemáticas, Universidad Nacional de Colombia, Bogotá D.C., Colombia*

[b] *Department of Mathematics and Institute for Scientific Computation (ISC), Texas A&M University, College Station, TX 77843-3368, USA*

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ABSTRACT

In this paper we consider the numerical upscaling of the Brinkman equation in the presence of high-contrast permeability fields. We develop and analyze a robust and efficient Generalized Multiscale Finite Element Method (GMsFEM) for the Brinkman model in two dimensions. In the fine grid, we use mixed finite element method with the velocity and pressure being continuous piecewise quadratic and piecewise constant finite element spaces, respectively. Using the GMsFEM framework we construct suitable coarse-scale spaces for the velocity and pressure that yield a robust mixed GMsFEM. We develop a novel approach to construct a coarse approximation for the velocity snapshot space and a robust small offline space for the velocity space. The stability of the mixed GMsFEM and a priori error estimates are derived. A variety of two-dimensional numerical examples are presented to illustrate the effectiveness of the algorithm.

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1. Introduction

In this paper, we design and analyze an efficient numerical method based on the generalized multiscale finite element method (GMsFEM) framework for Brinkman type system of partial differential equations in the context of mixed finite element method. The Brinkman equation is widely accepted in the mathematical modeling of flows in heterogeneous fields, e.g., vuggy carbonate reservoirs, low porosity filtration devices and biomedical hydrodynamic studies [1,2]. In these applications, the simple Darcy model is only capable of modeling slow flow problems [3,4], thus Darcy flow is not suitable to cavity problems. Moreover, even though the Darcy–Stokes interface model is capable to describe the flow of a viscous fluid in cavity and porous media, it is not practically feasible since the precise information about the location and geometry of the interface between vugs and the porous matrix is inaccessible, neither are the experimentally determined values related to the interface conditions. The Brinkman flow behaves like a Darcy flow and a Stokes flow for regions with very large permeability values and with small permeability values, respectively. Hence, in comparison with the popular Stokes–Darcy interface model, the Brinkman model can describe both a Stokes and a Darcy flow without using a complex interface condition. Therefore, the accuracy and efficiency of the Brinkman flow simulation is of significant practical interest [5–8]. In our earlier work [9], we derived homogenization results for high-contrast Brinkman flow in a periodic permeability field. We showed that the homogenization method can simplify the high-contrast periodic Brinkman model, and the resulting solution is a good approximation of the original Brinkman model.

* Corresponding author.
E-mail address: jcgalvisa@unal.edu.co (J. Galvis).

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In this work, we investigate the high-contrast Brinkman flow in general high-contrast permeability fields instead of the periodic fields as analyzed in [9]. In most porous media related flow problems, there always exists a coefficient that can vary several orders of magnitude within the analysis domain. This type of media is usually referred as a high-contrast medium. Often, model reduction techniques are required for efficiently resolving such multiscale problems. These techniques all rely on a coarse grid approximation, obtained by discretizing the problem on a coarse grid, much coarser than the fine grid, and a suitable coarse-grid formulation of the problem. In the literature, several different approaches have been proposed to obtain the coarse-grid formulation, which can be roughly divided into upscaling models [10,11] and multiscale methods (see, e.g., [12–18] and the references therein). Among existing multiscale methods, the GMsFEM framework [15,19] of recent origin has demonstrated great promise; see [15,20–22,18,23,19] for methodological developments and extensive applications. In the GMsFEM, the coarse grid problem is obtained by locally constructing reduced order models for the solution space on coarse regions and then employing a global formulation on the resulting reduced space.

The Brinkman model can be written as

$$\nabla p - \mu \Delta u + \kappa^{-1} u = f \quad \text{in } \Omega,$$
$$\operatorname{div} u = 0 \quad \text{in } \Omega,$$

where p is the fluid pressure, u represents the velocity, f denotes the forcing term and $\Omega \in R^n$, $n = 2, 3$ with polyhedral boundary. Here, μ is the viscosity and $\kappa = \kappa(x)$ is a heterogeneous multiscale coefficient that models the permeability of the porous medium. We assume that the variations of κ occur within a very fine scale and therefore a direct simulation of this model is costly.

As mentioned above, one of the main advantages of the Brinkman model is that it can capture Stokes and Darcy type flow behavior depending on the value of κ without the usage of a complex interface condition as needed in the Stokes–Darcy interface model. This is very convenient when modeling complicated porous scenarios such as a vuggy medium. However, this advantage of the Brinkman model does not come for free: it brings the challenge of effectively designing numerical homogenization or upscaling methodologies since the resulting upscaling method must capture the correct flow behavior in corresponding regions. This difficulty increases in the case of high-contrast coefficients due to the fact that, in a single coarse region, the permeability field can have variations of several order of magnitude that make it difficult to compute effective parameters for the permeability or boundary conditions using classical multiscale finite element methods.

In this work, we develop an efficient (multiscale) solver based on the GMsFEM framework [19] for the Brinkman flow in heterogeneous high-contrast permeability fields. As far as we know, there has not been any paper on the GMsFEM of Brinkman flow in high-contrast flow. GMsFEM is based on MsFEM, and it is an enrichment of MsFEM in the sense that it adds more basis in each coarse neighborhood to approximate the high-contrast problem more accurately. Upscaling approaches are another type of model reduction method derived from homogenization. In this framework, as in many other multiscale model reduction techniques, one divides the computation into two stages, i.e., the offline stage and the online stage. In the offline stage, a reduced dimension space is constructed, and it is then used in the online stage to construct multiscale basis functions. These multiscale basis functions can be re-used for any input parameter to solve the problem on a coarse grid. The main idea behind the construction of the offline and online spaces is to design appropriate local spectral-based selection of important modes that generate the snapshot space. In [19], several general strategies for designing the local spectrum-based selection procedures were proposed. Here, we focus on the generation of snapshots spaces, and rigorous convergence analysis of the resulting coarse approximation. Further, we establish stability estimate of the mixed GMsFEM (in the form of inf–sup conditions) for the proposed reduced dimension spaces. The convergence analysis extends that for elliptic equations with high-contrast coefficients [15].

We present four numerical examples to illustrate the performance of the proposed approach. One challenge with the Brinkman model is the construction of a stable finite element discretization, and lots of researchers are working on it [8,24]. In this paper, we apply the simple $Q2\backslash Q0$ element for the sake of simplicity since the focus of our work is on the stability of the GMsFEM for Brinkman flow. We study the model with different high-contrast parameter κ^{-1}. As we know that the velocity of the flow is slower in a region with a higher parameter κ^{-1}. If the velocity is slow in one part of the region, we refer this region as a slow region and the flow as a slow flow, similar to a faster region and a faster flow. When the value of the parameter κ^{-1} is very small (of order 1), then the flow in this region behaves as the Stokes flow, and we refer it as a free flow. Our first test is a faster Darcy flow going through the slower region; the second is a slower Darcy flow past a faster Darcy flow regions; the third is a free flow going across the Darcy flow region; and the last test is a Darcy flow passing the free flow region. All the numerical results indicate that the proposed GMsFEM is robust and accurate.

The rest of the paper is organized as follows. In Section 2, we present preliminaries on the Brinkman model and the GMsFEM. The construction of the coarse spaces for the GMsFEM is displayed in Section 3. In Section 4, numerical results for several representative examples are showed. The proofs of our main results, including stability and a priori error estimates, are exhibited in Section 5. Finally, we conclude our paper with some remarks in Section 6.

2. Preliminaries

In this section, we first present in detail the Brinkman problem we are dealing with, and the corresponding fine-scale discretization of this problem. Then we discuss briefly the multiscale strategy of solving this problem.

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