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A constrained matrix least-squares problem in structural dynamics model updating

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HIGHLIGHTS

- All necessary constraints are considered in our model.
- Alternating projection method is first used to update the finite element model.
- Numerical results are compared with the recent results.

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ABSTRACT

A constrained matrix least-squares problem in structural dynamics model updating is considered in this paper. Desired matrix properties, including the satisfaction of the linear equation, symmetric positive semidefiniteness and sparsity, are imposed as side constraints. Alternating projection method is applied to solve the constrained minimization problem. The results of the numerical examples show that the proposed method works well.

1. Introduction

In this paper, we consider the following constrained matrix least-squares problem

$$\begin{aligned} \min \frac{1}{2}\|X - X_0\|_F^2 \\ \text{s.t. } XA = B, \\ X = X^T,\ X \geq 0, \\ \text{sparse}(X) = \text{sparse}(X_0), \end{aligned} \tag{1}$$

where $A, B \in R^{n\times m} (m \leq n)$ and $X_0 \in R^{n\times n}$ are given matrices, $X \geq 0$ means X is positive semidefinite, $\text{sparse}(X) = \text{sparse}(X_0)$ denotes the sparsity requirement on X and $\| \bullet \|_F$ stands for the Frobenius norm of a matrix.

Problem (1) arises typically in the finite element model updating in structural dynamics [1–3]. Let $M_a, K_a \in R^{n\times n}$ and n be the analytical mass matrix, stiffness matrix and the number of degrees of freedom of the finite element model, respectively. The analytical model of a real-life structure, obtained by the finite element technique, may be represented by the generalized eigenvalue problem

$$K_a x = \lambda M_a x,$$

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where λ and x are the eigenvalue and corresponding eigenvector. Due to the complexity of the engineering structures, the finite element models often fail to reproduce the dynamic behavior of the actual structures accurately. Hence, the finite element model should be updated and have definite physical meaning, such as the satisfaction of the dynamics equation, the nonnegativeness of energy and the structural connectivity. Such a procedure can be mathematically formulated as problem (1).

In the past decades, special versions of problem (1) and its applications have received considerable discussions. Problems of finding the best approximation to a given matrix subject to different kinds of linear constraints, symmetry and positive semidefiniteness are discussed in [4–15]. Problems containing zero/nonzero pattern constraint and linear constraints can be found in [16–19,5,20,21]. Recently, Yuan [22] considered problem (1) using subgradient algorithm for two different models.

Notice that problem (1) is a minimization of a strictly convex quadratic function over the intersection of a finite collection of closed convex sets in $R^{n\times n}$, we employ the alternating projection method to solve problem (1) which was firstly proposed by Von Neumann [23] to find the projection of a given point in Hilbert space onto the intersection of two closed subspaces. Cheney and Goldstein [24] extended Von Neumann's result to two closed convex sets. Dykstra et al. [25,26] adjusted Von Neumann's method slightly to ensure the convergence of the algorithm. Glunt et al. [27] applied alternating projection method on the approximation of distance matrices. Using Dykstra's modified method, Escalante and Raydan [28] solved matrix least-squares problem subject to symmetric positive definiteness constraint, box constraint and given pattern constraint. Smith [29] discussed Dykstra's alternating projection method in model updating problem to preserve sparsity, symmetry and positive definiteness of property matrices. Using the same method, Moreno, Datta and Raydan [30] solved symmetry preserving matrix model updating problem. However, at least one constraint of problem (1) is ignored.

Throughout this paper, the following notations will be used. For $A \in R^{n\times m}$, A^+ denotes the Moore–Penrose generalized inverse of A, $P_S(A)$ represents the projection of A onto a closed convex set S, and specially, A_+ is the orthogonal projection of A onto the positive semidefinite cone. I_n is the $n \times n$ identity matrix. This paper is organized as follows. In Section 2, we discuss the conditions ensuring the feasible region of problem (1) is nonempty. In Section 3, the alternating projection method for problem (1) is presented. In Section 4, three numerical experiments are performed to illustrate the efficiency of the proposed method. Conclusions are given in Section 5.

2. The feasible region

Denote the feasible region of problem (1) by D. Define

$$S_1 = \{X \in R^{n\times n} | XA = B, X^T = X,\ X \geq 0\} \quad \text{and}$$
$$S_2 = \{X \in R^{n\times n} | \text{sparse}(X) = \text{sparse}(X_0)\}.$$

It is obvious that $D = S_1 \cap S_2$. In this section, we deduce some conditions ensuring that the feasible region D is nonempty.

Lemma 2.1 ([31]). *Let $E \in R^{n\times n}$, $F \in R^{n\times k}$, $G \in R^{k\times k}$, $H = \begin{pmatrix} E & F \\ F^T & G \end{pmatrix}$. If $H^T = H$, then $H \geq 0$ if and only if*

$$E \geq 0, \quad G - F^T E^+ F \geq 0, \quad \text{and} \quad \text{rank}(E, F) = \text{rank}(E). \tag{2}$$

Lemma 2.2. *Let $A, B \in R^{n\times m} (m \leq n)$. Assume that the singular value decomposition (SVD) of the matrix A is*

$$A = U \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} V^T, \tag{3}$$

where $U = [U_1, U_2] \in R^{n\times n}$, $V = [V_1, V_2] \in R^{m\times m}$ are orthogonal matrices, $\Sigma = diag(\sigma_1, \ldots, \sigma_r) > 0$, $r = rank(A)$, $U_1 \in R^{n\times r}$ and $V_1 \in R^{m\times r}$. Then the matrix equation

$$XA = B \tag{4}$$

has a symmetric positive semidefinite solution if and only if

$$BV_2 = 0, \qquad U_1^T BV_1 \Sigma^{-1} = \Sigma^{-1} V_1^T B^T U_1 \geq 0, \quad \text{and} \tag{5}$$
$$\text{rank}(U_1^T BV_1 \Sigma^{-1}, \Sigma^{-1} V_1^T B^T U_2) = \text{rank}(U_1^T BV_1 \Sigma^{-1}), \tag{6}$$

in which case the solution $X \in R^{n\times n}$ is

$$X = U \begin{pmatrix} U_1^T BV_1 \Sigma^{-1} & \Sigma^{-1} V_1^T B^T U_2 \\ U_2^T BV_1 \Sigma^{-1} & G + U_2^T BV_1 \Sigma^{-1} (U_1^T BV_1 \Sigma^{-1})^+ \Sigma^{-1} V_1^T B^T U_2 \end{pmatrix} U^T, \tag{7}$$

where $G \in R^{(n-r)\times(n-r)}$ is an arbitrary symmetric positive semidefinite matrix.

Proof. If Eq. (4) has a symmetric positive semidefinite solution X, left-multiplying two sides of Eq. (4) by A^T yields $A^T XA = A^T B$. Since $A^T XA$ is symmetric positive semidefinite, then so is $A^T B$. From Eqs. (3) and (4), one has $XU_1 \Sigma V_1^T = B$. It follows