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Proximal operator of quotient functions with application to a feasibility problem in query optimization

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ABSTRACT

In this paper we determine the proximity functions of the sum and the maximum of componentwise (reciprocal) quotients of positive vectors. For the sum of quotients, denoted by Q_1, the proximity function is just a componentwise shrinkage function which we call q-shrinkage. This is similar to the proximity function of the ℓ_1-norm which is given by componentwise soft shrinkage. For the maximum of quotients Q_∞, the proximal function can be computed by first order primal–dual methods involving epigraphical projections.

The proximity functions of Q_ν, $\nu = 1, \infty$ are applied to solve convex problems of the form $\operatorname{argmin}_x Q_\nu(\frac{Ax}{b})$ subject to $x \geq 0$, $\mathbf{1}^\top x \leq 1$. Such problems are of interest in selectivity estimation for cost-based query optimizers in database management systems.

1. Introduction

This work is motivated by query optimization in database management systems (DBMSs) where the optimal query execution plan depends on the accurate estimation of the proportion of tuples, called selectivities, that satisfy the predicates in the query. Models for selectivity estimation as those in [1] require the solution of a feasibility problem. More precisely, based on an under-determined linear system of equations $Ax = b$ which has no nonnegative solution $x \geq 0$ we are looking for a 'correct' right-hand side $\hat{b}$ such that a nonnegative solution exists. There exists a large amount of literature on feasibility problems, see [2,3] and the references therein. In particular we refer to the SMART algorithm connection with minimizing the Shannon entropy [4,5]. However, our approach is different from the known ones with respect to the functional which has to be minimized. By the results in [6] there is a strong evidence that in query optimization it is the (reciprocal) quotients of the components $\max\{\frac{\hat{b}_i}{b_i}, \frac{b_i}{\hat{b}_i}\}$ which should be made small, not their differences. In this paper we are interested in the sum of such quotients denoted by Q_1 and their maximum Q_∞.

Recently, first order primal–dual methods were successfully applied in data processing, see, e.g., the overview papers [7,8] and the references therein. These methods are based on splitting methods known in optimization theory for a long time. In this paper we are interested in applying first order primal–dual methods as an alternative to second order cone programming for solving problems involving the quotient functionals Q_ν, $\nu = 1, \infty$. Basically, these iterative algorithms decouple the

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problem into different proximation problems and the success of the method depends on the efficient solution of these proximation problems. Therefore, we examine the proximity function of our quotient functionals Q_ν, $\nu = 1, \infty$ first. We show that the proximity function of the sum of quotients, Q_1, is a componentwise shrinkage function which we call q shrinkage. It is slightly more involved than the componentwise soft-shrinkage which is the proximity function of the ℓ_1-norm since one has to solve a third order equation. The proximity function of the maximum of quotients Q_∞ can be computed by an alternating minimization method of multipliers which involves componentwise epigraphical projections. These componentwise steps can be computed in parallel.

We apply our findings to solve the feasibility problem described above and demonstrate the results obtained by different error measures by a numerical example.

The outline of this paper is as follows: In Section 2 we introduce the quotient distance between positive numbers and use it to define quotient functionals of vectors with positive components. In Section 3 we determine the proximity operator of the quotient functionals. We use our findings in Section 4 for solving feasibility problems appearing, e.g., in selectivity estimations which are necessary for query optimization in DBMSs. We describe the selectivity estimation problem, propose primal–dual minimization algorithms and demonstrate the performance by a numerical example. Conclusions are drawn in Section 5.

2. Quotient functions

The function $q\colon (0, +\infty) \times (0, +\infty) \to [0, +\infty)$ defined by

$$q(x, y) := \frac{\max(x, y)}{\min(x, y)},$$

can be considered as a 'distance function'. It is symmetric in its components and since

$$q(x, y) - 1 = \frac{|x - y|}{\min(x, y)},$$

it fulfills $q(x, y) - 1 = 0$ if and only if $x = y$. Clearly, the quotient distance does not fulfill a triangle inequality. A relative of $q(x, y)$, the so-called *generalized relative distance*, given for $(x, y) \in \mathbb{R}^* \times \mathbb{R}^*$ by

$$\frac{|x - y|}{\max(x, y)},$$

has been used in [9–12]. For a relation between the generalized relative error and the quotient distance we refer to [13]. Due to its *zero-homogeneity*

$$q(\lambda x, \lambda y) = q(x, y), \quad \lambda > 0 \tag{1}$$

the quotient distance is used as a contrast measure in image processing [14].

For fixed $b > 0$, we generalize $q(\cdot, b)$ to the whole real axis by $q(\cdot, b)\colon \mathbb{R} \to [0, +\infty]$ with

$$q(x, b) := \begin{cases} \dfrac{x}{b} & \text{if } b \le x, \\ \dfrac{b}{x} & \text{if } 0 < x < b, \\ +\infty & \text{otherwise.} \end{cases} \tag{2}$$

The function $q(\cdot, b)$ is convex and continuous. Moreover, we have by (1) that $q(x, b) = q(x/b, 1)$. We will write just q instead of $q(\cdot, 1)$. Note that for positive arguments the function $\log q(\cdot, b) = |\log b - \log(\cdot)|$ is neither convex nor concave.

In the following, set $\mathcal{I}_N := \{1, \ldots, N\}$. For fixed $b = (b_k)_{k=1}^N \in (0, +\infty)^N$ we are interested in the quotient functionals $Q_1(\cdot, b), Q_\infty(\cdot, b) : \mathbb{R}^N \to [0, +\infty]$ defined by

$$Q_1(x, b) := \sum_{k=1}^{N} q(x_k, b_k) \quad \text{and} \quad Q_\infty(x, b) := \max_{k \in \mathcal{I}_N} q(x_k, b_k). \tag{3}$$

We set $Q_\nu := Q_\nu(\cdot, 1)$, $\nu \in \{1, \infty\}$. In the following, norms $\|\cdot\|$ are Euclidean norms.

3. Proximity operator of quotient functionals

Let $\Gamma_0(\mathbb{R}^N)$ denote the space of proper, convex and lower semi-continuous functions on $\mathbb{R}^N$ mapping to $\mathbb{R} \cup \{+\infty\}$. For a function $\varphi \in \Gamma_0(\mathbb{R}^N)$ and $\gamma > 0$, the *proximal function* $\mathrm{prox}_{\gamma\varphi} : \mathbb{R}^N \to \mathbb{R}^N$ is defined by

$$\mathrm{prox}_{\gamma\varphi}(x) := \operatorname*{argmin}_{t \in \mathbb{R}^N} \; \varphi(t) + \frac{1}{2\gamma}\|x - t\|^2.$$