Journal of Computational and Applied Mathematics 287 (2015) 32–43

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Uniformly asymptotic behavior of ruin probabilities in a time-dependent renewal risk model with stochastic return[☆]

Yang Yang[a,b,*], Zhimin Zhang[c], Tao Jiang[d], Dongya Cheng[e]

[a] *Department of Statistics, Nanjing Audit University, Nanjing, 211815, China*
[b] *School of Economics and Management, Southeast University, Nanjing, 210096, China*
[c] *Department of Statistics and Actuarial Science, Chongqing University, 401331, China*
[d] *School of Statistics and Mathematics, Zhejiang Gongshang University, 310018, China*
[e] *School of Mathematical Sciences, Soochow University, Suzhou, 215006, China*

ARTICLE INFO

Article history:
Received 27 August 2014
Received in revised form 14 March 2015

MSC:
91B30
60G51
60K05

Keywords:
Ruin probability
Dependence
Lévy process
Dominated variation
Uniformity
Simulation

ABSTRACT

In this paper, we consider a time-dependent risk model, where an insurance company is allowed to invest its wealth in financial assets and the price process of the investment portfolio is described as a geometric Lévy process. When claim sizes have dominatedly varying tails, we obtain some asymptotic formulae for ruin probabilities holding uniformly for some finite or infinite time horizons. We further perform some simulations to check the accuracy of our formulae.

1. Introduction

Consider a time-dependent risk model. The claim sizes $X_1, X_2, \ldots$ are a sequence of independent and identically distributed (i.i.d.) nonnegative random variables (r.v.s) with common distribution $F = 1 - \overline{F}$. The inter-arrival times $\theta_1, \theta_2, \ldots$ form another sequence of i.i.d. nonnegative r.v.s with common distribution G. Denote by X and θ the generic r.v.s of the claim sizes and inter-arrival times. Assume that $(X_1, \theta_1), (X_2, \theta_2), \ldots$ are i.i.d. random vectors, and there exists some dependence structure between X and θ. Suppose that the claim arrival times $\tau_n = \sum_{k=1}^{n} \theta_k,\ n = 1, 2, \ldots$ constitute a renewal counting process

$$N(t) = \sup\{n \geq 0 : \tau_n \leq t\}, \quad t \geq 0,$$

☆ Research supported by National Natural Science Foundation of China (Nos. 71471090, 11471058, 71171177), the Humanities and Social Sciences Foundation of the Ministry of Education of China (No. 14YJCZH182), China Postdoctoral Science Foundation (Nos. 2014T70449, 2012M520964), Natural Science Foundation of Jiangsu Province of China (No. BK20131339), Qing Lan Project, PAPD, Project of Construction for Superior Subjects of Statistics of Jiangsu Higher Education Institutions.

* Corresponding author at: Department of Statistics, Nanjing Audit University, Nanjing, 211815, China.
E-mail address: yyangmath@gmail.com (Y. Yang).

http://dx.doi.org/10.1016/j.cam.2015.03.020
0377-0427/

which describes the number of claims up to time t, with a finite mean function $\lambda(t) = EN(t) \to \infty$ as $t \to \infty$. Suppose that the insurer is allowed to make risk-free and risky investments. The price process of the investment portfolio is described as a geometric Lévy process $e^{R_t},\ t \geq 0$ with $R_t,\ t \geq 0$ being a Lévy process, which starts from zero and has independent and stationary increments. Assume that $R_t,\ t \geq 0$ is independent of $(X_1, \theta_1), (X_2, \theta_2), \ldots$. This assumption on price processes is widely used in mathematical finance. One can refer to Paulsen [1], Paulsen and Gjessing [2], Wang and Wu [3], Kalashnikov and Norberg [4], Cai [5], Yuen et al. [6] and Yuen et al. [7], among others. In such a risk model, the stochastic present value of aggregate claims up to time $t \geq 0$ can be expressed as

$$D_t = \sum_{k=1}^{\infty} X_k e^{-R_{\tau_k}} \mathbf{1}_{\{\tau_k \leq t\}}, \tag{1.1}$$

where $\mathbf{1}_A$ denotes the indicator function of an event A. Then, for any $t \geq 0$, the discount value of the surplus process with stochastic return on investments of an insurance company is described as

$$U(t) = x + \int_{0-}^{t} c(s) e^{-R_s} \mathrm{d}s - D_t, \tag{1.2}$$

where $x \geq 0$ is the initial risk reserve of the insurance company, and $c(t)$ denotes the density function of premium income at time t. Throughout the paper, we assume that the premium density function $c(t)$ is bounded, i.e., $0 \leq c(t) \leq M$ for some constant $M > 0$ and all $t \geq 0$.

In the actuarial literature, the ruin probabilities are defined to be the probability that the surplus falls below zero. Precisely speaking, for any $t \geq 0$, the finite-time ruin probability is

$$\Psi(x, t) = \mathrm{P}\Big(\inf_{s \in [0,t]} U(s) < 0 \Big| U(0) = x\Big),$$

and the infinite-time ruin probability is

$$\Psi(x, \infty) = \mathrm{P}\Big(\inf_{s \geq 0} U(s) < 0 \Big| U(0) = x\Big).$$

In the present paper, we shall investigate the asymptotics for ruin probabilities holding uniformly for all t such that $\lambda(t)$ is positive. For this purpose, as in [8], define $\Lambda(t) = \{t : 0 < \lambda(t) \leq \infty\} = \{t : \mathrm{P}(\theta_1 \leq t) > 0\}$ with $\underline{t} = \inf\{t : \lambda(t) > 0\} = \inf\{t : \mathrm{P}(\theta_1 \leq t) > 0\}$. Clearly,

$$\Lambda = \begin{cases} [\underline{t}, \infty], & \text{if } \mathrm{P}(\theta_1 = \underline{t}) > 0; \\ (\underline{t}, \infty], & \text{if } \mathrm{P}(\theta_1 = \underline{t}) = 0. \end{cases}$$

For the above risk model with a constant premium rate $c > 0$ (i.e., $c(t) = c$ for all $t > 0$) and a constant interest force $\delta > 0$ (e.g., $R_t = \delta t$ for all $t > 0$), if the claim sizes $X_1, X_2, \ldots$, the inter-arrival times $\theta_1, \theta_2, \ldots$ and the Lévy process $R_t,\ t \geq 0$ are mutually independent, some earlier works on ruin probabilities can be found in [9–12], among others; Tang [8] and Hao and Tang [13] derived some uniform results for all Λ. Some recent results on dependent claim sizes or dependent inter-arrival times have also been obtained by many researchers, such as Chen and Ng [14], Yang and Wang [15], Wang et al. [16].

Recently, Li [17] considered another type of dependent renewal risk model, in which $(X_1, \theta_1), (X_2, \theta_2), \ldots$ are assumed to be i.i.d. copies of a generic pair (X, θ) with dependent components X and θ, see Assumption A for details. This dependence structure was proposed by Asimit and Badescu [18], and was thoroughly studied by Li et al. [19]. Such a dependent risk model has been widely studied because in almost all kinds of insurance, there exists some dependence between the claim size and inter-arrival time. Think that, if the deductible retained to insureds is raised, then the inter-arrival time will increase because small claims will be ruled out; while the likelihood of a large claim will increase if the claim size is new-worse-than-used. Meanwhile, an advantage of this model is that independence among the increments of the surplus process over claim arrival times is preserved.

The main goal of this paper is to investigate the uniformly asymptotic behavior for the tail probability $\mathrm{P}(D_t > x)$ and the ruin probability $\Psi(x, t)$ in the above time-dependent risk model with dominatedly varying tailed claim sizes. Our obtained results extend the corresponding ones in [17]. Meanwhile, we also perform some simulations to check the accuracy of the asymptotic formulae for finite-time and infinite-time probabilities.

The rest of the paper is organized as follows. In Section 2 we state the main result of this paper after introducing some necessary preliminaries. Section 3 presents the proof of the main result after a series of lemmas. Section 4 performs some simulations to verify the approximate relationships in the main result.

2. Preliminaries and main result

Hereafter, all limit relationships hold for x tending to ∞ unless stated otherwise. For two positive functions $a(x)$ and $b(x)$, we write $a(x) \sim b(x)$ if $\lim a(x)/b(x) = 1$; write $a(x) \lesssim b(x)$ if $\limsup a(x)/b(x) \leq 1$; write $a(x) \gtrsim b(x)$ if $\liminf a(x)/$