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Assessing the reliability of general-purpose Inexact Restoration methods[☆]

E.G. Birgin[a,*], L.F. Bueno[b], J.M. Martínez[c]

[a] *Department of Computer Science, Institute of Mathematics and Statistics, University of São Paulo, São Paulo, SP, Brazil*

[b] *Institute of Science and Technology, Federal University of São Paulo, São José dos Campos, SP, Brazil*

[c] *Department of Applied Mathematics, Institute of Mathematics, Statistics, and Scientific Computing, State University of Campinas, Campinas, SP, Brazil*

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A B S T R A C T

Inexact Restoration methods have been proved to be effective to solve constrained optimization problems in which some structure of the feasible set induces a natural way of recovering feasibility from arbitrary infeasible points. Sometimes natural ways of dealing with minimization over tangent approximations of the feasible set are also employed. A recent paper [Banihashemi and Kaya (2013)] suggests that the Inexact Restoration approach can be competitive with well-established nonlinear programming solvers when applied to certain control problems without any problem-oriented procedure for restoring feasibility. This result motivated us to revisit the idea of designing general-purpose Inexact Restoration methods, especially for large-scale problems. In this paper we introduce affordable algorithms of Inexact Restoration type for solving arbitrary nonlinear programming problems and we perform the first experiments that aim to assess their reliability. Initially, we define a purely local Inexact Restoration algorithm with quadratic convergence. Then, we modify the local algorithm in order to increase the chances of success of both the restoration and the optimization phase. This hybrid algorithm is intermediate between the local algorithm and a globally convergent one for which, under suitable assumptions, convergence to KKT points can be proved.

1. Introduction

Inexact Restoration (IR) is an attractive approach for solving Nonlinear Programming problems, see [1–12]. The idea of IR methods is that, at each iteration, feasibility and optimality are addressed in different phases. In the Restoration Phase the algorithms aim to improve feasibility and in the Optimization Phase they aim to improve optimality, preserving a linear approximation of feasibility. These algorithms have been successfully used in applications in which there exists a natural way to improve (or even obtain) feasibility (see [1,6,9,10] among others).

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* Corresponding author.
E-mail addresses: egbirgin@ime.usp.br (E.G. Birgin), lfelipebueno@gmail.com (L.F. Bueno), martinez@ime.unicamp.br (J.M. Martínez).

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In [9,10] control problems of the following form were considered:

$$\begin{aligned} &\text{Minimize} \quad \int_{t_0}^{t_f} f_0(s(t), u(t))\, dt \\ &\text{subject to} \quad \dot{s}(t) = F(s(t), u(t)) \\ &\qquad\qquad\quad s(t_0) = s_0, \end{aligned} \tag{1}$$

where the state variable is $s(t) \in \mathbb{R}^{n_s}$, $\dot{s} = ds/dt$, the control variable is $u(t) \in \mathbb{R}^{n_u}$, t varies between t_0 and t_f, $f_0 : \mathbb{R}^{n_s} \times \mathbb{R}^{n_u} \to \mathbb{R}$, and $F : \mathbb{R}^{n_s} \times \mathbb{R}^{n_u} \to \mathbb{R}^{n_s}$. The initial state is given by $s_0 \in \mathbb{R}^{n_s}$. The time domain $[t_0, t_f]$ is subdivided into N intervals with equidistant points $t_i = t_{i-1} + \Delta t$ or, equivalently, $t_i = t_0 + i\ \Delta t,\ i = 1, \ldots, N$, where $\Delta t = (t_f - t_0)/N$ and, hence, $t_N = t_f$. Considering the Euler discretization scheme $s_{i+1} = s_i + \Delta t F(s_i, u_i)$ and approximating the integral in the objective function of (1) by its Riemann sum, we arrive to the discretized optimal control problem

$$\begin{aligned} &\text{Minimize} \quad \Delta t \sum_{i=0}^{N-1} f_0(s_i, u_i) \\ &\text{subject to} \quad s_{i+1} = s_i + \Delta t F(s_i, u_i), \quad i = 0, \ldots, N-1, \end{aligned} \tag{2}$$

where s_0 is given, the variables s_i approximate the states $s(t_i)$ for $i = 1, \ldots, N$, and the variables u_i approximate the controls $u(t_i)$ for $i = 0, \ldots, N-1$. The number of variables is $n = (n_s + n_u)N$ and the number of (equality) constraints is $m = n_s N$. Higher-order discretization schemes such as the ones in the Runge–Kutta family of methods can be used. In the case of problem (2), restoration consists of fixing the control variables and approximately solving the initial value problem. In the Optimization Phase, IR methods change both the control and the state variables. The restoration procedure is quite natural and, so, it is not surprising the obtention of good numerical results using IR approaches.

Surprisingly, in a recent paper, Banihashemi and Kaya [13] applied an IR scheme to a family of control problems for which the natural initial-value restoration procedure cannot be applied anymore and obtained better results with their method than with a standard well-established nonlinear optimization software. Although the problems addressed in [13] possess an interesting particular structure, the algorithm used for recovering feasibility does not exploit that structure at all. Therefore, we found the relative efficiency reported in [13] surprising. The Banihashemi–Kaya paper motivated us to revisit the application of IR to general nonlinear programming problems, without regarding any specific structure. The main question is: Is it worthwhile to develop a universal constrained optimization package based on the IR idea? In the present paper we wish to report the first steps in the process of answering this question and developing the corresponding software.

Global convergence theories for modern Inexact Restoration methods were given in [12,11,5,7,14,15]. In [12] the theory is based on trust regions and a quadratic penalty merit function. The trust-region approach employing a sharp Lagrangian as merit function was introduced in [11]. In [5,7] global convergence was based on a filter approach. Fischer and Friedlander [14] proved global convergence theorems based on line searches and exact penalty functions. A global convergence approach that employs the sharp Lagrangian and line searches was defined in [15]. Local and superlinear convergence of an Inexact Restoration algorithm for general problems was proved in [2] and a general local framework that includes composite-step methods was given in [16]. In the present paper we adopt the scheme of [2] that requires improvement of feasibility with controlled distance to the current point at the feasibility phase. Here this requirement will be achieved minimizing the distance to the current point subject to the minimization of the quadratic approximation of infeasibility.

We will define four algorithms. The first one will be a local method, similar to the method introduced in [2], for which local quadratic convergence will be proved under suitable sufficient conditions. (In [2] sufficient conditions for the welldefinedness of the algorithm were not provided.) The second method will be a variation of the local method that aims to improve the global convergence performance and has the same local convergence properties as the first one. The third method uses the basic tools of the first two but is globally convergent thanks to the employment of line searches and sharp Lagrangians, as in [15]. The fourth one is a hybrid combination of the second and the third methods, designed to improve its computational performance.

We wish to provide a practical assessment of the reliability of IR methods on *general* (potentially large-scale) Nonlinear Programming. For this purpose some decisions will be taken on the concrete implementation of each particular IR method, leaving apart the degrees of freedom that the general approach provides. In particular, the first trial point for the feasibility phase will come from the solution of a quadratic box-constrained problem and the first trial point of the optimization phase will come from the solution of a feasible quadratic programming problem. The implementation of the four algorithms introduced in this paper will be described and a comparison between them and against well established Nonlinear Programming solvers will be provided. As a final consequence we will establish a conclusion about the reliability of using IR ideas for general problems, in which specific characteristics of the feasible set or the objective function are not used at all.

We will consider the problem

$$\text{Minimize}\, f(x) \text{ subject to } h(x) = 0, \quad x \in \Omega \tag{3}$$

where $f : \mathbb{R}^n \to \mathbb{R}$, $h : \mathbb{R}^n \to \mathbb{R}^m$, and $\Omega = \{x \in \mathbb{R}^n \mid \ell \le x \le u\}$. For all $x \in \Omega$ and $\lambda \in \mathbb{R}^m$ we define the Lagrangian $L(x, \lambda)$ by

$$L(x, \lambda) = f(x) + \sum_{i=1}^{m} \lambda_i h_i(x).$$