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Almost sure exponential stability of the backward Euler–Maruyama scheme for stochastic delay differential equations with monotone-type condition

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ABSTRACT

This paper is a continuation of our previous paper, in which, the second author, with Mao and Szpruch examined the almost sure stability of the Euler–Maruyama (EM) and the backward Euler–Maruyama (BEM) methods for stochastic delay differential equations (SDDEs). In the previous results, although the drift coefficient may defy the linear growth condition, the diffusion coefficient is required to satisfy the linear growth condition. In this paper we want to further relax the condition. Under monotone-type condition, this paper will give the almost sure stability of the BEM for SDDEs whose both drift and diffusion coefficients may defy the linear condition. This improves the existing results considerably.

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1. Introduction

One of the important characteristics of the SDDEs is stability. The fact that SDDEs in most cases cannot be solved explicitly has been the main motivation for the development of different approximate methods. There is extensive concerned with moment stability of various numerical methods (see [1–7]). There is also papers to discuss almost sure stability (see, for example, [8–12]). But most of these given their result with linear condition or linear growth condition.

This paper is a continuation of the articles [13] by the second author with Mao and Szpruch, and of [14] by Mao, Shen and Alison, in which the almost sure stability of the numerical solutions to stochastic differential equations (SDEs) as well as SDDEs is established. But in these existing results in [13,14], although the drift coefficient may defy the linear growth condition, the diffusion coefficient is still required to satisfy the linear growth condition. This excludes many important classes of stochastic systems, for example, the well-known stochastic Lotka–Volterra model, [15], exhibits a similar type of non-linearity

$$dx(t) = \mathrm{diag}(x_1, x_2, \dots, x_n(t))[(b + Ax^2(t))dt + x(t)dw(t)].$$

The only results we know, where the stability of the numerical approximations was considered for super-linear diffusion is [16,17]. Mao and Szpruch proved asymptotic stability of implicit numerical methods in polynomial growth setting (see [16]). More recently, Huang studied the mean-square stability of two classes of theta method for SDDEs under monotone

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condition. In [17], Huang proved the split-step theta method is exponentially mean square stable; but only obtained asymptotic stability of the classical stochastic theta method which becomes BEM method if $\theta = 1$. To the best of our knowledge there is no result about exponential stability of classical BEM approximation in super-linear diffusion setting.

The main aim of this paper is to improve the existing results and admit that both the drift and diffusion coefficients of stochastic differential equations may defy the linear growth condition and examine their almost sure exponential stability of classical BEM method. Consider the n-dimensional SDDE of the form

$$dx(t) = f(x(t), x(t-\tau), t)dt + g\,(x(t), x(t-\tau), t)\,dw(t), \quad t \geq 0, \tag{1}$$

with initial data $\xi \in \mathbf{C}^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$, $f, g \colon \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}^n$, are Borel measurable, and $w(t)$ is a scalar Brownian motion.

Under monotone-type condition, in which both the drift and diffusion coefficients may defy the linear growth condition, this paper will address the following questions:

(Q1) If the exact solution of (1) is almost surely exponentially stable, can its numerical approximation preserve this stability?
(Q2) Can we use the large stepsize to obtain the above result?
(Q3) How to calculate the speed of the exact solution and its numerical approximation converge to the trivial solution. And what is the relationship between these two speed?

For the (Q1), we shall focus on the BEM method, under monotone-type condition give the positive answer. For the (Q2), we only need a weak restriction of step size. Indeed if the BEM method is well defined, we do not need any other conditions in the proof. For the (Q3), we give a feasible method to calculate the speed and show that the BEM method will reproduce the decay rate for sufficiently small step size.

The rest of the paper is arranged as follows. In Section 2, we introduce some necessary notations which are used in the sequel and present conditions. Eq. (1) almost surely admits a unique global solution and this solution is almost surely exponentially stable under the above conditions. In Section 3, we will establish the almost sure exponential stability of the BEM approximation under the monotone-type condition. We will give a further result in Section 4. We show that the BEM method is almost sure exponential stability in polynomial growth setting, which is more convenient to check than monotone-type condition. The last section gives numerical examples and simulations to illustrate our conclusion.

2. Notations and preliminaries

Throughout this paper, unless otherwise specified, we use the following notations. Let $|\cdot|$ be the Euclidean norm in $\mathbb{R}^n$. If A is a vector or matrix, its transpose is denoted by A^T. If A is a matrix, its Hilbert–Schmidt norm is denoted by $|A| = \sqrt{\mathrm{trace}(A^T A)}$. Let $\mathbb{R}_+ = [0, \infty)$, and $\tau > 0$. Denoted by $\mathbf{C}([-\tau, 0]; \mathbb{R}^n)$ the family of continuous functions from $[-\tau, 0]$ to $\mathbb{R}^n$. Let $\mathbf{C}^b_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable bounded $\mathbf{C}([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \xi(\theta) : -\tau \leq \theta \leq 0$. The inner product of $X, Y \in \mathbb{R}^n$ is denoted by $\langle X, Y \rangle$ or $X^T Y$. If $a, b \in \mathbb{R}$, $a \vee b$ denotes the maximum of a and b and $a \wedge b$ represents their minimum. For notational simplicity, *const* denotes a generic positive constant, whose precise values may be different for different appearances.

Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions, that is, it is right continuous and increasing while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets. Let $w(t)$ be a scalar Brownian motion defined on this probability space.

In this paper we choose the Lyapunov function $V(x) = |x|^2$ and define a corresponding operator $\mathcal{L}V : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}$ by

$$\mathcal{L}V(x, y, t) = 2x^T f(x, y, t) + |g(x, y, t)|^2. \tag{2}$$

If $x(t)$ is a solution of Eq. (1), by the Itô formula,

$$dV(x(t)) = \mathcal{L}V(x(t), y(t), t)dt + 2x(t)^T g(x(t), y(t), t)dw(t). \tag{3}$$

For the stability purpose of this paper, assume $f(0, 0, t) \equiv 0$ and $g(0, 0, t) \equiv 0$. This implies that Eq. (1) admits a trivial solution $x(t, 0) \equiv 0$. As a standing hypothesis, let us impose the following local Lipschitz condition (see [18,19]) on the coefficients f and g.

Assumption 2.1. Both f and g satisfy the local Lipschitz condition. That is, for each integer $j = 1, 2, \ldots$, there exists a constant $C_j > 0$ such that

$$|f(x, y, t) - f(\bar{x}, \bar{y}, t)| \vee |g(x, y, t) - g(\bar{x}, \bar{y}, t)| \leq C_j(|x - \bar{x}| + |y - \bar{y}|),$$

for all $t \geq 0$ and those $x, \bar{x}, y, \bar{y} \in \mathbb{R}^n$ with $|x| \vee |\bar{x}| \vee |y| \vee |\bar{y}| \leq j$.

Replacing the classical linear growth condition or one-side linear growth condition, this paper needs the following monotone condition.

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