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Convex optimization over fixed point sets of quasi-nonexpansive and nonexpansive mappings in utility-based bandwidth allocation problems with operational constraints

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ABSTRACT

Network bandwidth allocation is a central issue in modern communication networks. The main objective of the bandwidth allocation is to allocate an optimal bandwidth for maximizing a predefined utility over the capacity constraints to traffic sources. When a centralized operator, which manages all the bandwidth allocations in the network, has a certain operational policy, the bandwidth allocation reflecting the operational policy should result in the network being more stable and reliable. Accordingly, we need to solve a network bandwidth allocation problem under both capacity constraints and operational constraints. To develop a novel algorithm for solving the problem, we translate the network bandwidth allocation problem into one of minimizing a convex objective function over the intersection of the fixed point sets of certain quasi-nonexpansive and nonexpansive mappings and propose a fixed point optimization algorithm for solving it. We numerically compare the proposed algorithm with the existing algorithm for solving a concrete bandwidth allocation problem and show its effectiveness.

1. Introduction

1.1. Background

Network resource allocation is needed for making communication networks reliable and stable, and it is of practical importance to allocate, fairly and effectively, finite network resources, such as power [1,2], channel [3], and bandwidth [4–8], to network users.

The objective of *utility-based bandwidth allocation* [6–8] in particular is to share the available bandwidth among traffic sources so as to maximize the overall utility under the capacity constraints.

The utility is modeled as a function, denoted by $\mathcal{U}$, of the transmission rates allocated to the traffic sources, and it represents the efficiency and fairness of bandwidth sharing [6–8]. We assume that $\mathcal{U}$ is continuously differentiable and concave. A well-known utility function is the *weighted proportionally fair* function [6–8] defined for all $x := (x_1, x_2, \ldots, x_S)^T \in \mathbb{R}_+^S \setminus \{0\}$ by $\mathcal{U}_{\mathrm{pf}}(x) := \sum_{s \in \mathcal{S}} w_s \log x_s$, where x_s (> 0) denotes the transmission rate of source s $(\in \mathcal{S} := \{1, 2, \ldots, S\})$, w_s (> 0) stands for the weighted parameter for source s, and $\mathbb{R}_+^S := \{(x_1, x_2, \ldots, x_S)^T \in \mathbb{R}^S : x_s \geq 0 (s \in \mathcal{S})\}$. The optimal bandwidth allocation corresponding to $\mathcal{U}_{\mathrm{pf}}$ is said to be *weighted proportionally fair*.

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The capacity constraint for each link is an inequality constraint in which the sum of the transmission rates of all the sources sharing the link is less than or equal to the capacity of the link, and hence, the capacity constraint set for each link $l\ (\in \mathcal{L} := \{1, 2, \ldots, L\})$ is expressed as $\mathbb{R}_+^S \cap C_l$, where

$$C_l := \left\{ x := (x_1, x_2, \ldots, x_S)^T \in \mathbb{R}^S : \sum_{s \in \mathcal{S}} x_s I_{s,l} \leq c_l \right\},$$

$c_l (> 0)$ stands for the capacity of link l, and $I_{s,l}$ takes the value 1 if l is the link used by source s, and 0 otherwise.

Therefore, our objective in bandwidth allocation is to solve the following *utility-based bandwidth allocation problem* [6], [8, Chapter 2] for maximizing the utility function subject to the capacity constraints:

$$\text{Maximize } \mathcal{U}_{\text{pf}}(x) \text{ subject to } x \in C,$$

where $C\ (\subset \mathbb{R}^S)$ stands for the capacity constraint set defined by

$$C := \mathbb{R}_+^S \cap \bigcap_{l \in \mathcal{L}} C_l = \mathbb{R}_+^S \cap \bigcap_{l \in \mathcal{L}} \left\{ (x_1, x_2, \ldots, x_S)^T \in \mathbb{R}^S : \sum_{s \in \mathcal{S}} x_s I_{s,l} \leq c_l \right\}. \quad (1)$$

1.2. Utility-based bandwidth allocation problem with operational constraint

We will discuss a utility-based bandwidth allocation problem subject to not only the capacity constraints but also an operational constraint. The operator has an operational policy to make the network more stable and reliable. For example, when sources exist in the network such that they get a low (resp. high) degree of satisfaction, the operator attempts to re-allocate bandwidth so as to enable them to send data at high (resp. low) transmission rates. When the available bandwidth is limited in the network, the operator needs to control the sum of the transmission rates of all sources. When the network is controlled by using a certain indicator function which represents the network's performance, the operator tries to design the network so as to satisfy a constraint incorporating the indicator function. The operational constraint set representing such operational policies can be written as

$$C_{\text{op}} := \left\{ x := (x_1, x_2, \ldots, x_S)^T \in \mathbb{R}^S : \mathcal{P}(x) \leq p \right\}, \quad (2)$$

where $\mathcal{P} : \mathbb{R}^S \to \mathbb{R}$ is convex (i.e., $\mathcal{P}$ satisfies the continuity [9, Theorem 4.1.3]) and is not always differentiable, and $p \in \mathbb{R}$. The operator can set $C_{\text{op}} = \{x \in \mathbb{R}^S : x_{s_0} \leq p\}$ when it tries to limit the transmission rate of source s_0, $C_{\text{op}} = \{x \in \mathbb{R}^S : \sum_{s \in \mathcal{S}} \omega_s x_s \leq p\}$ $(\omega_s \geq 0\ (s \in \mathcal{S}))$ when it tries to limit the transmission rates of all sources, and $C_{\text{op}} = \{x \in \mathbb{R}^S : \sum_{s \in \mathcal{S}} \omega_s \mathcal{P}_s(x_s) \leq p\}$ $(\omega_s \geq 0\ (s \in \mathcal{S}),\ \mathcal{P}_s : \mathbb{R} \to \mathbb{R}$ is nondifferentiable[1]) when the network is controlled by $\mathcal{P}(x) := \sum_{s \in \mathcal{S}} \omega_s \mathcal{P}_s(x_s)$.

Therefore, we can formulate a utility-based bandwidth allocation problem with both the capacity constraints and the operational constraint as follows:

$$\text{Maximize } \mathcal{U}_{\text{pf}}(x) \text{ subject to } x \in C \cap C_{\text{op}}, \quad (3)$$

where one assumes $C \cap C_{\text{op}} \neq \emptyset$.[2]

There are useful methods [10–16] for solving optimization problems with nonsmooth constraints and optimization problems with nonsmooth objective functions. One avenue for addressing the lack of smoothness is via a variety of smoothing techniques (e.g., deterministic smoothing techniques [10] and convolution-based smoothing techniques [16]). Other methods are, for example, path search algorithms [12, Subchapter 8.1], trust region methods [12, Subchapter 8.4], equation-based algorithms [12, Chapter 9], variational inequality-based algorithms [12, Chapter 10], subgradient methods [14, Chapter 3], and bundle trust region algorithms [14, Subchapter 3.3].

To develop an algorithm for solving Problem (3), we will focus on the following *variational inequality* [17, Chapter II], [11, Chapter 1], [18, Chapter I], [19, Subchapter 6.D] which coincides with Problem (3) [17, Chapter 2, Proposition 2.1 (2.1) and (2.2)].

Problem 1.1 (*Utility-Based Bandwidth Allocation Problem Under Capacity Constraints*).

$$\text{Find } x^\star \in \text{VI}\left(C \cap C_{\text{op}}, -\nabla \mathcal{U}_{\text{pf}}\right) := \left\{ x^\star \in C \cap C_{\text{op}} : \langle x - x^\star, -\nabla \mathcal{U}_{\text{pf}}(x^\star) \rangle \geq 0 (x \in C \cap C_{\text{op}}) \right\},$$

where $\langle \cdot, \cdot \rangle$ stands for the inner product of $\mathbb{R}^S$ and $\nabla \mathcal{U}_{\text{pf}} : \mathbb{R}^S \to \mathbb{R}^S$ is the gradient of $\mathcal{U}_{\text{pf}}$.

In this paper, we shall devise an iterative algorithm for solving Problem 1.1 based on iterative techniques for *optimization over the fixed point sets* of certain mappings. With this goal in mind, we will translate Problem 1.1 into an optimization problem over the intersection of the fixed point sets.

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[1] If the network's performance increases when all sources' transmission rates are more than a certain value x^0 (> 0), $\mathcal{P}_s(x)$ is expressed as 0 $(0 \leq x \leq x^0)$, or $x - x^0$ $(x \geq x^0)$.

[2] For example, $0 \in C \cap C_{\text{op}}$ holds when $\mathcal{P}(x) := \sum_{s \in \mathcal{S}} \omega_s x_s$, $\omega_s \in \mathbb{R}$ $(s \in \mathcal{S})$, and $p \geq 0$. Since the operator knows the explicit form of C, it can set C_{op} such that $C \cap C_{\text{op}} \neq \emptyset$.