



## Stress strength reliability in the presence of fuzziness

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## ABSTRACT

This paper investigates the stress–strength reliability in the presence of fuzziness. The fuzzy membership function is defined as a function of the difference between stress and strength values, and the fuzzy reliability of single unit and multicomponent systems are calculated. The inclusion of fuzziness in the stress–strength interference enables the user to make more sensitive analysis. Illustrations are presented for various stress and strength distributions.

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## 1. Introduction

Stress–strength analysis has attracted a great deal of attention in reliability literature. In a traditional setup of stress–strength analysis, a unit or system operates as soon as its strength exceeds the stress imposed upon it. Most of the studies in this setup focus on the computation and estimation of the reliability for various stress and strength distributions such as exponential, Weibull, normal, and gamma. A comprehensive review of the topic is presented in [1]. Some recent discussions in this direction are in [2–6].

Stress–strength reliability has also been studied under multi-component setup, i.e. the system consists of more than two components. Bhattacharya and Johnson [7] considered the  $k$ -out-of- $n$  structure under stress strength setup. According to their definition, the system consists of  $n$  components, and functions if at least  $k$  components survive a common random stress. Hanagal [8] studied the series system reliability under stress–strength setup. The stress–strength reliability of a consecutive  $k$ -out-of- $n$  system has been studied in [9]. Recent works on multi-component stress strength reliability are in [10–13].

In this paper, we study the stress–strength reliability in the presence of fuzziness which is attached to the difference between stress and strength values. In particular, we follow the idea of Huang [14] who investigated the reliability analysis in the presence of fuzziness attached to operating time of a system. The paper is organized as follows. In Section 2, we define and study the fuzzy stress–strength reliability for a single unit system. Section 3 extends the results to a multicomponent system having an arbitrary structure.

## 2. Reliability evaluation and estimation

Let  $X$  and  $Y$  denote respectively the strength and stress random variables. If  $X$  and  $Y$  are independent with respective distribution functions  $F_X$  and  $F_Y$ , then the traditional stress–strength reliability can be computed from

$$R = P\{X > Y\} = \iint_{x>y} dF_X(x)dF_Y(y). \quad (1)$$

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The conventional stress–strength reliability given by (1) can be rewritten as

$$R = P\{X > Y\} = \int_0^\infty \int_0^\infty C_{A(y)}(x) dF_X(x) dF_Y(y),$$

where the characteristic function for the set  $A(y) = \{x : x > y\}$  is given as

$$C_{A(y)}(x) = \begin{cases} 0, & \text{if } x \leq y \\ 1, & \text{if } x > y. \end{cases}$$

For the fuzzy event “ $X$  is fuzzily bigger than  $Y$ ” (denoted by  $X \succ Y$ ), let  $\mu_{A(y)}(x)$  denote the corresponding membership function. Then from the definition of fuzzy probability [15], the fuzzy stress–strength reliability can be represented as

$$R_F = P\{X \succ Y\} = \int_0^\infty \int_0^\infty \mu_{A(y)}(x) dF_X(x) dF_Y(y), \quad (2)$$

where we define the appropriate membership function as

$$\mu_{A(y)}(x) = \begin{cases} 0, & \text{if } x \leq y \\ h(x - y), & \text{if } x > y \end{cases}$$

for an increasing function  $h$ . Thus Eq. (2) can be rewritten as

$$R_F = P\{X \succ Y\} = \int_0^\infty \int_y^\infty h(x - y) dF_X(x) dF_Y(y). \quad (3)$$

Eq. (3) assigns a value for the reliability by considering the difference between strength and stress values. The traditional stress–strength reliability only considers the event that  $X$  is greater than  $Y$ . In the fuzzy case, the reliability considers the distance  $X - Y$  when  $X$  is greater than  $Y$ . According to this new fuzzy stress–strength interference, for  $X = x$  and  $Y = y$ , with an increase in the values of  $x - y$  the system becomes more reliable. Therefore, such a consideration may enable us to make a more sensitive analysis.

Below we illustrate the computation of  $R_F$  when stress and strength distributions are exponential.

**Example 1.** Let  $F_X(x) = 1 - e^{-\lambda_1 x}$ ,  $x > 0$  and  $F_Y(y) = 1 - e^{-\lambda_2 y}$ ,  $y > 0$ . Suppose that  $h(u) = 1 - e^{-ku}$ ,  $u > 0$ , i.e. the corresponding membership function is

$$\mu_{A(y)}(x) = \begin{cases} 0, & \text{if } x \leq y \\ 1 - e^{-k(x-y)}, & \text{if } x > y. \end{cases}$$

Then

$$\begin{aligned} R_F &= \int_0^\infty \int_y^\infty (1 - e^{-k(x-y)}) \lambda_1 e^{-\lambda_1 x} \lambda_2 e^{-\lambda_2 y} dx dy \\ &= \frac{\lambda_2}{\lambda_1 + \lambda_2} - \frac{\lambda_1 \lambda_2}{(\lambda_1 + k)(\lambda_1 + \lambda_2)}. \end{aligned}$$

It is easy to see that  $R_F$  tends to conventional reliability  $R = \frac{\lambda_2}{\lambda_1 + \lambda_2}$  as  $k \rightarrow \infty$ .

The computation of  $R_F$  can be difficult for more complex distributions  $F_X$  and  $F_Y$ . In this case, it might be appropriate to get bounds for the fuzzy reliability  $R_F$ .

Let  $h(u) = 1 - g(u)$  and  $g(u)$  be a convex function. Then one can show that (see Appendix)

$$R_F \leq R \cdot [1 - g(E(X - Y | X > Y))]. \quad (4)$$

The conditional expected value  $E(X - Y | X > Y)$  can be more easily calculated than (3).

**Example 2.** Let  $X \sim N(\mu_X, \sigma_X^2)$  and  $Y \sim N(\mu_Y, \sigma_Y^2)$ . Then it can be shown that

$$E(X - Y | X > Y) = \mu_X - \mu_Y + \frac{1}{1 - \Phi\left(\frac{\mu_Y - \mu_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right)} \sqrt{\frac{\sigma_X^2 + \sigma_Y^2}{2\pi}} e^{-\frac{(\mu_X - \mu_Y)^2}{2(\sigma_X^2 + \sigma_Y^2)}},$$

and

$$R = P(X > Y) = P(X - Y > 0) = 1 - \Phi\left(\frac{\mu_Y - \mu_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}\right),$$

where  $\Phi(z)$  is the cdf of standard normal variate. Thus an upper bound for the fuzzy reliability can be obtained using the last two equations in (4). In Table 1, we compute the upper bound  $R_F^u$  for different values of  $k$  when  $\mu_X = 3$ ,  $\sigma_X = 0.5$ ,  $\mu_Y = 1$ ,  $\sigma_Y = 1$  and  $h(u) = 1 - g(u) = 1 - e^{-ku}$ . We also compute  $R_F$  by the method of computer simulation.

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