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Optimizing equiareality of NURBS surfaces using composite Möbius transformations

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ABSTRACT

The equiareality of NURBS surfaces greatly affects the results of visualization and tessellation applications, especially when dealing with extruding and intruding shapes. To obtain more equiareal parameterizations of NURBS surfaces than the Möbius based method, an optimization algorithm is presented in this paper based on the more flexible composite reparameterization functions. For fixed knots, the optimal composite reparameterization can be obtained using the Levenberg–Marquardt method. For a given tolerance, a uniform subdivision scheme is interleaved with the optimization procedure and this process finishes until the change of the equiareal deviation energy is less than the given tolerance. Examples are given to show the performance of our algorithm for visualization and tessellation applications.

1. Introduction

NURBS surfaces play an increasingly important role in contemporary Computer Aided Design (CAD). The results of most surface algorithms such as surface registration, surface visualization, surface tessellation, and so on [1–5], highly depend on the surface parameterization while the surface modification and surface fitting in reverse engineering both may introduce resultant surfaces with improper parameterizations. One of the reasons why the designers prefer NURBS surfaces lies in their easy human interactions. To generate a target geometric shape, the control points and their weights are adjusted manually by the designer or computed by some auxiliary algorithms, which may destroy some desirable properties of the surface parameterization and affect the subsequent surface manipulations. At the same time, an explicit surface representation with control points and weights is preferred by most of the above mentioned Computer-Aided Design applications. How to improve parameterization of given NURBS surfaces while keeping an explicit surface representation is the focus of this paper.

A NURBS surface has an intrinsic rational polynomial mapping from the 3D surface to the 2D parameter domain. By surface reparameterizations [6,1–3], the surface may have infinitely many different parameterizations. Depending on where and how it will be used, one may need to find a suitable or optimal parameterization out of the infinitely many, or to convert the given parameterization into another (more) suitable one [6,1–3]. In many applications, such as the quantitative analysis and visualization of brain, joint bone and mechanical surfaces with rich extruding and intruding features, it is highly desirable that the parameterization can preserve area elements of the 3D surfaces, so that many area-related patterns,

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e.g., intestinal density, activation extent, thickness, etc., can be better represented in the parameter domain [5]. Equiareality is a natural and direct 2D surface extension of arc-length parameterizations for curve cases, which play an important role for many CAD applications and have been studied extensively in [7–18]. Moreover, an equiareal parameterization will lead to more robust and stable computations for derivative based algorithms such as surface intersection, curvature computation, and so on [19,20].

In the past twenty years, how to achieve optimal parameterization of Bézier curves has been studied extensively in the literature such as [7,8,10–18]. Farouki [8] identified arc-length parameterization as the optimal parameterization of Bézier curves. By minimizing an integral which measures the deviation from arc-length parameterization, the optimal representation is obtained by solving a quadratic equation. Jüttler [10] presented a simplified approach to Farouki's result by using a back substitution in the integral. Costantini et al. [7] obtained closer approximations to the arc-length parameterization by applying composite reparameterizations to Bézier curves. Farin [21] showed that for a circular arc represented by its rational quadratic representation, its parameterization is a chord length one. Motivated by Farin's work, Lu [22] identified a family of curves that can be parameterized by chord length.

For triangle meshes and subdivision surfaces, we concentrate on analyzing the difference between their parameterization and freeform surface parameterization, a detailed review of mesh and subdivision surface parameterization techniques being beyond the scope of this article. For the details of mesh parameterization, the reader is recommended to see the survey paper by Floater and Hormann [9] and the references therein. For the details of subdivision surface parameterization, see the papers by He et al. [23,24] for the latest progress. The parameterization of triangle meshes has been studied extensively in the last decade and still remains as a hot topic until now [9,25–31,5]. The main purpose of the research on the parameterization of triangle meshes is to construct a suitable, bijective mapping between the triangle mesh embedded in 3D and a simple 2D domain, referred to as the parameter space or parameter domain. To minimize the parameterization distortion in either angles or areas, many different algorithms have been proposed in the literature [9,25–31,5]. Also He et al. presented algorithms for parameterizing subdivision surfaces in [23,24].

As the NURBS surface already has a rational polynomial mapping from the 2D parameter domain (a rectangle) to the 3D surface, its parameterization has some specific properties different from the parameterization of triangle meshes and subdivision surfaces. First the NURBS surface has an intrinsic mapping already and we do not need to construct an initial surface mapping from the 3D surface to the 2D parameter domain, which is the case for triangle meshes and subdivision surfaces. Second the parameterization of NURBS surface is a continuous rational polynomial mapping while those of triangle meshes and subdivision surfaces are discrete, usually defined by the correspondence between their vertices and the correspondence of points inside the triangles/quads is obtained from the vertices correspondence by interpolation techniques. If we convert the NURBS surface into a triangle mesh and apply the mesh parameterization method, some parameterization results can be obtained subsequently. However, there is one main drawback for this kind of methods. The resultant surface representation is not NURBS anymore, which is problematic for subsequent CAD algorithms designed for freeform surfaces. Though we can reconstruct the NURBS surface by traditional least-square fitting methods from the triangle parameterization, neither the surface shape nor the triangle parameterization are preserved precisely during the fitting procedure, which is not allowed for high accurate CAD applications.

While most of the successes have been reported about the parameterization of NURBS curves, triangle meshes and subdivision surfaces [7–9,23,24,10,11,26,12–18,5], the NURBS surface parameterization has not met with similar achievements. The results of rendering and tessellation applications for NURBS surfaces largely depend on the parameterization quality. He et al. [24] gave a rational bicubic reparameterization method to improve the parameterization of the approximate Gregory patches such that the new parameterization conforms better to that of the given subdivision surface. Both the explicit representation of the reparameterized surface and the equiareality of the final surface are not considered therein. Yang et al. [1] presented an algorithm to optimize the uniformity of iso-parameter curves for Bézier surfaces based on Möbius transformations [32,1], which can change only the distribution of iso-parameter curves but not the shape of them. However, the above method [1] minimizes the uniformity only on sampled iso-parameter curves, not on the whole surface. To improve the equiareality of NURBS surfaces, Yang et al. [33] presented an optimization algorithm based on the Möbius transformation. The optimal Möbius transformation is obtained by computing the intersection of two planar algebraic curves, whose coefficients are computed explicitly for Bézier and B-spline surface, while numerically for NURBS surfaces. However the attractive simplicity of the algorithm in [33] is compromised by the limited ability of Möbius transformations to achieve sufficiently close approximations to equiareal parameterizations for practical applications (see Fig. 1).

The aim of this paper is to solve the optimization problem of minimizing the equiareality of NURBS surfaces with a more powerful composite Möbius transformation (see Fig. 2). The initial set of knots of the composite Möbius transformation is determined by a uniform sampling method. The explicit optimal composite transformations are then obtained on the fixed set of knots that does not alter the surface degree by the Levenberg–Marquardt method. The set of knots is refined repeatedly until the improvement of the equiareal deviation energy is less than the user-specified tolerance. Compared with the method presented in [33], our method can obtain a much more equiareal surface parameterization while keeping the intrinsic geometry and smoothness of the surface. Although the resulting parameterization is only C^0 with respect to the new parameter, it does not in any way alter the smoothness of the surface. Furthermore, since the optimization procedure typically brings the surface very close to being equiareal across the surface, the jumps in parametric speed incurred at the nodes of the C^0 re-parameterization function are usually insignificant.