Journal of Computational and Applied Mathematics 279 (2015) 13–30

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Optimal control of mean-field jump-diffusion systems with delay: A stochastic maximum principle approach

Qingxin Meng [a], Yang Shen [b,*]

[a] Department of Mathematical Sciences, Huzhou University, Zhejiang 313000, China

[b] School of Risk and Actuarial Studies and CEPAR, UNSW Business School, The University of New South Wales, Sydney, NSW 2052, Australia

ARTICLE INFO

Article history:
Received 13 December 2013
Received in revised form 11 October 2014

Keywords:
Mean-field
Stochastic delay differential equation
Advanced backward stochastic differential equation
Optimal control
Stochastic maximum principle

ABSTRACT

This paper is concerned with an optimal control problem under mean-field jump-diffusion systems with delay. Firstly, some existence and uniqueness results are proved for a jump-diffusion mean-field stochastic delay differential equation and a jump-diffusion mean-field advanced backward stochastic differential equation. Then necessary and sufficient maximum principles for control systems of mean-field type and with delay are established under certain conditions. A mean-field, delayed, linear-quadratic control problem is finally discussed using the obtained maximum principles.

1. Introduction

The stochastic maximum principle is one of the major approaches to solve optimal control problems. [1,2] pioneered the research on the stochastic maximum principle and extended Pontryagin's deterministic maximum principle. Subsequent works in this topic include [3–8], to just name a few. Simply speaking, the stochastic maximum principle reduces solving an optimal control problem to solving a forward–backward stochastic differential equation. An optimal control is then attained by maximizing the Hamiltonian function.

In recent years, the systems with interacting behavior have attracted increasing attention in the stochastic control theory. The so-called mean-field models are designed to study such systems. The history of the mean-field models can be dated back to the early works of [9,10]. Since then, the mean-field models have been found useful to describe the aggregate behavior of a large number of mutually interacting particles in diverse areas of physical sciences, such as statistical mechanics, quantum mechanics and quantum chemistry. Recent interest is to study the stochastic maximum principle under the mean-field models. Previous works include [11–17], and references therein.

Time delay is also a common feature in the real-world systems. One major reason is that many real-world systems evolve according to not only their current state but also essentially their previous history. For theory, examples and applications of the systems with time delay, interested readers may refer to [18]. Indeed, the phenomenon of past path-dependence exists in the fields of physics, chemistry, biology, finance, economics, etc. Therefore, it is of great relevance to develop the maximum principles under control systems with delay and apply these maximum principles to solve practical problems arising in the real world. See, for example, [16,17,19–27].

In this paper, we consider a jump-diffusion control system of mean-field type and with delay. Our aim is to establish necessary and sufficient maximum principles for such a system. We first prove the existence and uniqueness of solutions

* Corresponding author.
E-mail addresses: mqx@hutc.zj.cn (Q. Meng), skyshen87@gmail.com (Y. Shen).

http://dx.doi.org/10.1016/j.cam.2014.10.011
0377-0427/

to a jump-diffusion mean-field stochastic delay differential equation (MF-SDDE) and a jump-diffusion mean-field advanced backward stochastic differential equation (MF-ABSDE) as well as the related continuous dependence theorems. Under the convexity assumption of the control domain, we prove the necessary maximum principle. The sufficient maximum principle is established under additional convexity assumptions on the terminal cost and the Hamiltonian. To illustrate our results, we consider a mean-field, delayed, linear-quadratic control problem. Using the necessary and sufficient maximum principles, the optimal control strategy is given explicitly in a dual representation. Compared with [16], which also considered stochastic maximum principles under delayed mean-field stochastic differential equations, the control system in our paper is more general since both jumps and average delay terms are incorporated in the state equation and both pointwise and average delay terms are present in the cost functional. In addition, we establish the existence and uniqueness of solutions to the MF-SDDE and MF-ABSDE by the continuous dependence theorems. Furthermore, in our linear quadratic example, we discuss the existence and uniqueness of the optimal control. It is worthwhile to mention that the adjoint equation in this paper is a jump-diffusion MF-ABSDE and thus differentiates this paper from [17], where a system of three-coupled jump-diffusion BSDEs is adopted as the adjoint equation.

The rest of this paper is structured as follows. Section 2 introduces basic notation. In Section 3, we present some preliminary results of the jump-diffusion MF-SDDE and MF-ABSDE. Section 4 formulates the control problem. In Section 5, we give the main results of this paper, i.e. the necessary and sufficient maximum principles for the mean-field jump-diffusion system with delay. Section 6 discusses a linear-quadratic control problem as an application of the maximum principles. The final section concludes the paper.

2. Notation

In this section, we introduce basic notation to be used throughout this paper. The notation to be presented resembles to that adopted by [17]. Denote by $A^\top$ the transpose of a vector or matrix A, by $\mathrm{tr}(A)$ the trace of a square matrix A, by $\mathrm{diag}(A)$ the diagonal matrix with the elements of a vector A on the diagonal, by $\langle A, B\rangle := A^\top B$ the inner product of two vectors A, B, by $|A| := \sqrt{\mathrm{tr}(A^\top A)}$ the norm of a vector or a matrix A, by $\mathcal{B}(E)$ the Borel σ-field generated of any set E, and by 0 the zero scalar, vector or matrix of appropriate dimensions. In addition, we let K and C be two positive generic constants, which may take different values from line to line in this paper.

Let $\mathcal{T} := [0, T]$ denote a finite time index, where $T < \infty$. We consider a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ carrying a Brownian motion and a Poisson random measure. Suppose that the Brownian motion and the Poisson random measure are stochastically independent under $\mathbb{P}$. We equip $(\Omega, \mathcal{F}, \mathbb{P})$ with a right-continuous, $\mathbb{P}$-complete filtration $\mathbb{F} := \{\mathcal{F}_t | t \in \mathcal{T}\}$ generated by the Brownian motion and the Poisson random measure. Furthermore, we assume that $\mathcal{F}_T = \mathcal{F}$ and $\mathcal{F}_t = \mathcal{F}_0$, for $t < 0$. Denote by $\mathbb{E}[\cdot]$ the expectation under $\mathbb{P}$. To simplify our notation, we will denote by

$$\bar{\varphi} := \mathbb{E}[\varphi], \qquad \bar{\varphi}(t) := \mathbb{E}[\varphi(t)], \tag{1}$$

for any random variable φ or random process $\varphi(\cdot)$ unless otherwise stated. Let $\{W(t)|t \in \mathcal{T}\} := \{(W_1(t), W_2(t), \ldots, W_d(t))^\top | t \in \mathcal{T}\}$ be a d-dimensional, $(\mathbb{F}, \mathbb{P})$-Brownian motion. Let $\{N(dt, d\zeta)|(t, \zeta) \in \mathcal{T} \times \mathbb{R}_0\} := \{(N_1(dt, d\zeta), N_2(dt, d\zeta), \ldots, N_l(dt, d\zeta))^\top | \mathcal{T} \times \mathbb{R}_0\}$ be an l-dimensional Poisson random measure on the product measurable space $(\mathcal{T} \times \mathbb{R}_0, \mathcal{B}(\mathcal{T}) \otimes \mathcal{B}(\mathbb{R}_0))$, where $\mathbb{R}_0 := \mathbb{R} \setminus \{0\}$, with the compensator

$$\begin{aligned}
\pi(dt, d\zeta) &:= (\pi_1(dt, d\zeta), \pi_2(dt, d\zeta), \ldots, \pi_l(dt, d\zeta))^\top, \\
&= (\nu_1(d\zeta), \nu_2(d\zeta), \ldots, \nu_l(d\zeta))^\top dt \\
&= \nu(d\zeta)dt,
\end{aligned}$$

such that $\{(N - \pi)([0, t] \times A)|t \in \mathcal{T}\}$ is an $\mathbb{R}^l$-valued, $(\mathbb{F}, \mathbb{P})$-martingale for all $A \in \mathcal{B}(\mathbb{R}_0)$ satisfying $\nu_j(A) < \infty, j = 1, 2, \ldots, l$. Here $\nu(d\zeta)$ is the Lévy measure of the jump amplitude of the Poisson random measure, which is assumed to be a σ-finite measure on $\mathbb{R}_0$ such that

$$\int_{\mathbb{R}_0} (1 \wedge \zeta^2)\nu_j(d\zeta) < \infty, \quad j = 1, 2, \ldots, l.$$

Write the l-dimensional compensated Poisson random measure as

$$\begin{aligned}
\widetilde{N}(dt, d\zeta) &:= N(dt, d\zeta) - \pi(dt, d\zeta) \\
&= N(dt, d\zeta) - \nu(d\zeta)dt, \\
&= (N_1(dt, d\zeta) - \nu_1(d\zeta)dt, \ldots, N_l(dt, d\zeta) - \nu_l(d\zeta)dt)^\top.
\end{aligned}$$

Let H be some given Hilbert space. On the filtered probability space $(\Omega, \mathcal{F}, \mathbb{F}, \mathbb{P})$, we introduce the following spaces of random variables or random processes:

- $\mathcal{L}^2(\mathcal{F}_T; H)$: the space of H-valued, $\mathcal{F}_T$-measurable random variables φ such that $\mathbb{E}[|\varphi|^2] < \infty$;
- $\mathcal{S}^2_{\mathbb{F}}(a, b; H)$: the space of H-valued, $\mathbb{F}$-adapted càdlàg processes $\{\varphi(t)|a \le t \le b\}$ such that $|\varphi(\cdot)|^2_{\mathcal{S}^2_{\mathbb{F}}(a,b;H)} := \mathbb{E}[\sup_{a\le t\le b} |\varphi(t)|^2] < \infty$;