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A new operational approach for numerical solution of generalized functional integro-differential equations

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ABSTRACT

In this paper, a class of linear and nonlinear functional integro-differential equations are considered that can be found in the various fields of sciences such as: stress–strain states of materials, motion of rigid bodies and models of polymer crystallization. The operational collocation method with shifted Jacobi polynomial bases is applied to approximate the solution of these equations. In addition, some theoretical results are given to simplify and reduce the computational costs. Finally, some numerical examples are presented to demonstrate the efficiency and accuracy of the proposed method.

1. Introduction

Finding the analytical solutions of functional equations has been devoted attention of mathematicians' interest in recent years. Several methods are proposed to achieve this purpose, such as [1–7]. Functional differential and integro-differential equations are often used to model some problems with aftereffect in mechanics and the related scientific fields [8–10]. Also, many typical examples such as stress–strain states of materials, motion of rigid bodies and models of polymer crystallization can be found in Kolmanovskki and Myshkis' monograph [11]. These equations are usually difficult to solve even analytically. So, there are particular methods that have been presented for numerical solutions of the mentioned problems. For example: Doha and et al. have used a spectral collocation method to solve the generalized pantograph equations, [12]. In [13], the Taylor collocation method is presented for numerically solving functional Volterra–Fredholm integral equations. Wang and et al. have applied the Lagrange collocation method to solve the Volterra–Fredholm integral equations, [14]. A Legendre collocation method is used in [15] to solve functional differential equations. Sezer and et al. employed a Taylor method for solving the pantograph equations, [16]. Abazari and et al. have used differential transform method to solve nonlinear integro-differential equations with proportional delay, [17]. Erdem and et al. have presented a Bernoulli polynomial approach for solving a class of mixed integro-differential–difference equations, [18].

In this paper, the general formulations for Jacobi operational matrices of the integration and product are constructed on the interval [0, 1]. The main aim is to improve Jacobi operational matrices for the solving generalized functional integro-differential equations with mixed argument. For this purpose, first the matrix relations between Jacobi polynomials and

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Jacobi polynomials with mixed argument are determined. Then their matrix forms are substituted into the given equation. Finally, the resulted operational matrices together with the collocation method are applied to reduce the solution of the suggested problems to the solution of a system of algebraic equations. The proposed method is used to solve linear and nonlinear functional integro-differential equations with mixed argument. On the other hand, the goal is finding of solution $u(h(x))$, where $h(x)$ is a known function. Functional differential and integro-differential equations with proportional delays are usually referred to as pantograph equations. That is unknown function appears as $u(rx)$ in equation, where $r \in \mathbb{R}$. In here, not only pantograph equations are considered, ($h(x) = rx$), but it is encountered the cases that $h(x)$ is a known general function. Hence, two algorithms will be presented for solving this class of equations.

The remainder of this paper is organized as follows: The Jacobi polynomials and some of their properties are introduced in Section 2. In Section 3, Jacobi operational matrices of the integration and product and some theoretical results are presented. In Section 4, an efficient error estimator is recalled. In Section 5, Jacobi operational matrices are applied to solve several functional integro-differential equations. A conclusion is presented in Section 6.

2. Jacobi polynomials and their properties

The well-known Jacobi polynomials are defined on the interval $z \in [-1, 1]$, with weighted function $w^{(\alpha,\beta)}(z) = (1-z)^{\alpha}(1+z)^{\beta}$, and can be determined with the following recurrence formula:

$$P_{i+1}^{(\alpha,\beta)}(z) = A(\alpha,\beta,i)\,P_i^{(\alpha,\beta)}(z) + z\,B(\alpha,\beta,i)\,P_i^{(\alpha,\beta)}(z) - D(\alpha,\beta,i)\,P_{i-1}^{(\alpha,\beta)}(z), \quad i = 1, 2, \ldots, \tag{1}$$

where

$$A(\alpha,\beta,i) = \frac{(2i+\alpha+\beta+1)(\alpha^2-\beta^2)}{2(i+1)(i+\alpha+\beta+1)(2i+\alpha+\beta)},$$
$$B(\alpha,\beta,i) = \frac{(2i+\alpha+\beta+2)(2i+\alpha+\beta+1)}{2(i+1)(i+\alpha+\beta+1)},$$
$$D(\alpha,\beta,i) = \frac{(i+\alpha)(i+\beta)(2i+\alpha+\beta+2)}{(i+1)(i+\alpha+\beta+1)(2i+\alpha+\beta)},$$

and

$$P_0^{(\alpha,\beta)}(z) = 1, \qquad P_1^{(\alpha,\beta)}(z) = \frac{\alpha+\beta+2}{2}z + \frac{\alpha-\beta}{2}.$$

The orthogonality condition of Jacobi polynomials is

$$\int_{-1}^{1} P_j^{(\alpha,\,\beta)}(z)\,P_k^{(\alpha,\,\beta)}(z)\,w^{(\alpha,\,\beta)}(z)\,dz = h_k\,\delta_{jk},$$

where

$$h_k = \frac{2^{\alpha+\beta+1}\Gamma(k+\alpha+1)\Gamma(k+\beta+1)}{(2k+\alpha+\beta+1)k!\,\Gamma(k+\alpha+\beta+1)}.$$

The analytic form of Jacobi polynomials is given by

$$P_i^{(\alpha,\beta)}(z) = \sum_{k=0}^{i} \frac{(-1)^{(i-k)}\Gamma(i+\beta+1)\Gamma(i+k+\alpha+\beta+1)}{\Gamma(k+\beta+1)\Gamma(i+\alpha+\beta+1)\,(i-k)!\,k!}\left(\frac{1+z}{2}\right)^k, \quad i = 0, 1, \ldots.$$

For practical use of Jacobi polynomials on the interval $x \in [0, 1]$, it is necessary to shift the defining domain by means of the following change variable:

$$z = 2\,x - 1, \quad x \in [0, 1].$$

The shifted Jacobi polynomials in x are then obtained as follows:

$$P_{i+1}^{(\alpha,\beta)}(x) = A(\alpha,\beta,i)\,P_i^{(\alpha,\beta)}(x) + (2\,x-1)\,B(\alpha,\beta,i)\,P_i^{(\alpha,\beta)}(x) - D(\alpha,\beta,i)\,P_{i-1}^{(\alpha,\beta)}(x), \quad i = 1, 2, \ldots, \tag{2}$$
$$P_0^{(\alpha,\beta)}(x) = 1, \qquad P_1^{(\alpha,\beta)}(x) = \frac{(\alpha+\beta+2)(2\,x-1)}{2} + \frac{\alpha-\beta}{2}.$$

The orthogonality condition and shifted weighted function are respectively,

$$\int_{0}^{1} P_i^{(\alpha,\,\beta)}(x)\,P_j^{(\alpha,\,\beta)}(x)\,w^{(\alpha,\,\beta)}(x)\,dx = \theta_i\,\delta_{ij},$$

where

$$\theta_i = \frac{\Gamma(i+\alpha+1)\Gamma(i+\beta+1)}{(2i+\alpha+\beta+1)\,i!\,\Gamma(i+\alpha+\beta+1)},$$