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Accelerating the solution of a physics model inside a tokamak using the (Inverse) Column Updating Method

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ABSTRACT

Many physics problems can only be studied by coupling various numerical codes, each modeling a subaspect of the physics problem that is addressed. In most cases, the "brute force" technique of running the codes one after the other in a loop until convergence is reached requires excessive CPU time. The present paper illustrates that re-writing the coupling as a root-finding problem, to which a quasi-Newton method – here the (Inverse) Column Updating Method – can be applied, is useful to push down the computation time, at the expense of a very modest amount of supplementary programming. A simplified version of the set of codes commonly used to describe plasma heating by radio frequency waves in a tokamak plasma is adopted for illustrating the potential of the speed-up method. It consists of a wave equation as well as a Fokker–Planck velocity space diffusion and a radial energy diffusion model. It is shown that with this approach a substantial reduction in CPU time needed for convergence can be obtained.

1. Introduction

Many physics problems are too complicated and/or too CPU time consuming to be addressed by a single set of simultaneously solved equations. A typical example is the description of phenomena occurring on two or more differing time scales. Tackling the problem adopting the general set of equations forces one to bring the numerical time step down to the characteristic time τ_{fast} on which the fastest of the phenomena occurs, even if relevant slower processes happen on τ's many orders of magnitude different from τ_{fast}. Such a procedure yields unnecessarily long computation times. Analytically separating the time scales yields sets of coupled equations, one set for each of the time scales, allows to tackle the problem faster [1,2]. Intelligent schemes for solving the resulting coupled sets of equations allow to further reduce the computation time for a given prescribed accuracy. A particular example – a simplified version of which is considered in the present paper – is the fast, driven response of a tokamak plasma to an electromagnetic perturbation brought about by a radio frequency (RF) or ion cyclotron resonance heating (ICRH) antenna in a magnetized tokamak plasma, and the (net) effect this has on the plasma. This type of wave heating is instrumental in magnetic confinement thermonuclear fusion devices to bring plasmas to fusion relevant temperatures at which fusion spontaneously occurs. In present-day devices (having strong static magnetic fields of several Tesla to confine the charged particles), the direct, driven response rate to the external excitation is about 8 orders of magnitude faster than the macroscopic response of the plasma to this excitation. Hence, on the fast time scale equilibrium

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macroscopic quantities such as temperature and density can be assumed to be *constant* in time. On the slow time scale, the details of what happens during a driving cycle are irrelevant so the fast time scale effects can be considered as purely driven at a prescribed frequency – without transients – so that only the *net* effect over a driver period is of consequence. A scheme capturing the process of plasma heating by RF waves then looks as follows: The RF electric field *pattern* changes as a consequence of the density and temperature changes, while the density and temperature change as a consequence of the changing local power level brought about by the changing wave pattern. This allows a self consistent modeling of the wave heating physics.

The here adopted simplified problem (referred to as ''toy problem'') consists of a wave equation, a Fokker–Planck equation and a diffusion equation, all of which are simplified versions of corresponding equations treating the wave–particle and particle–particle interactions more rigorously. Each equation is solved separately using its own solver. The simple loop philosophy of the physics model (which was initially used to tackle the wave heating problem by brute force) is as follows:

(1) The computation is started from a given initial *temperature* profile. All other physical parameters and profiles are frozen in time and space (e.g. particle diffusion is not accounted for, i.e. the density is assumed not to be affected by the waves).

(2) Radio frequency waves are launched into a tokamak plasma using an antenna. These waves are evanescent close to the antenna and become propagative inside the plasma, where they are damped by collisionless damping. The energy locally deposited is redistributed in two ways:

   (a) For the heated minority species, the distribution function is modified – a high energy tail being formed – and the energy of the particles (the effective temperature) increases. The heated species interact with the other plasma constituents, thereby indirectly heating them. The balance between heating through wave–particle interaction and cooling through particle–particle Coulomb collisional interaction is described by the Fokker–Planck equation. For this toy problem only the Fokker–Planck equation of the minority is solved.

   (b) The electrons and majority ions indirectly receive power from the minority species through Coulomb collisions; due to the simplifications in the wave equation model direct heating mechanisms for these bulk species are excluded. The energy, the bulk species receive from the minority gas diffuses through the plasma. A diffusion equation is solved to track the fate of this energy, setting up a new temperature profile.

(3) The new and old temperature profiles are compared. If the changes are significant, the computation returns to (1); if not, the computation ends.

The speed-up realized in this paper adopts steps 2a, 2b and 3 but substitutes the simple loop by a more sophisticated scheme.

This paper is structured as follows: in Section 2 the different parts of the physics model are explained, in Section 3 we show how these can be solved and compare the different methods. We end with a short conclusion.

2. Description of the adopted model

In this section the three adopted equations are briefly sketched. The focus is on explaining the philosophy of the simplifications, and the role of the retained equation rather than on the actual derivation of the equations, all of which are standard. The interested reader can e.g. consult [3–5] for more details on the wave–particle and particle–particle interaction physics involved.

2.1. Wave equation

Maxwell's equations relate the electric and magnetic field to currents and charges. Eliminating the magnetic field yields an equation for the electric field,

$$\nabla \times \nabla \times \mathbf{E} = k_o^2\mathbf{E} + i\omega\mu_o[\mathbf{J}_{\text{antenna}} + \mathbf{J}_{\text{plasma}}], \tag{1}$$

in which $\bar{\bar{K}}\cdot\mathbf{E} = \mathbf{E}+i\mathbf{J}_{\text{plasma}}/\omega\epsilon_o$ with $\bar{\bar{K}}$ the dielectric tensor, ω the antenna frequency and where the $\vec{J}$'s are current densities, either flowing on the antenna or inside the plasma. When only the fast time scale driven motion is retained, the equation of motion can locally be solved and the driven velocity can be expressed in terms of the electric field. As $\vec{J}_{\text{plasma}} = \sum_\alpha q_\alpha\langle\vec{v}\rangle$ in which $\langle\vec{v}\rangle$ is the average wave-driven velocity (the sum is on the various types of species the plasma consists of), this yields a constitutive expression relating the plasma current and the electric field via which the dielectric tensor $\bar{\bar{K}}$ is defined.

Rather than solving the wave equation in 3 dimensions and for all 3 electric field components, only the dominant electric field components excited by a fast wave antenna will be retained and inhomogeneity of the macroscopic quantities is only allowed in one direction. The fast ''magnetosonic'' wave is the longest wavelength wave that can be sustained in the plasma; it is the temperature (and density) corrected vacuum wave characterized by a driven electric field almost perpendicular to the confining magnetic field $\vec{B}_o$ and by a magnetic field that is aligned with $\vec{B}_o$; the driven magnetic and electric field is related through Maxwell's equation $\nabla \times \vec{E} = i\omega\vec{B}$, where ω is the frequency of the antenna-driven oscillation. This wave carries its energy electromagnetically i.e. via the Poynting flux. The dominant component of the confining magnetic field in a tokamak is in the toroidal direction, identified here with the z-direction; the antenna is replaced by a current sheet with only a component in the y-direction, the latter representing the poloidal direction. As only the fast magnetosonic wave is excited, the electric field component parallel to the confining magnetic field can be omitted in a first approximation. Furthermore, the dielectric tensor will be truncated at its leading order finite temperature corrections. Assuming the z- and y-directions