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A new *a posteriori* parameter choice strategy for the convolution regularization of the space-fractional backward diffusion problem

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ABSTRACT

In this paper, we consider a backward space-fractional diffusion problem. We propose an *a posteriori* parameter choice rule for the regularization method given in Zheng and Wei (2010), where the authors proposed a regularization method called convolution regularization method, and gave an *a priori* parameter choice strategy. In this paper, we study the same problem but give a new *a posteriori* parameter choice based on a modified version of the discrepancy principle, and obtain a log-type error estimate under an additional source condition. Numerical results show that our method is feasible.

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1. Introduction

In this paper, we consider the following backward space-fractional diffusion problem

$$u_t(x,t) = {}_xD_\theta^\alpha(x,t), \quad x \in \mathbb{R},\ t \in (0,T), \tag{1.1}$$

$$u(x,T) = f(x), \quad x \in \mathbb{R}, \qquad u(x,t)|_{x\to\pm\infty} = 0, \tag{1.2}$$

where the space-fractional derivative ${}_xD_\theta^\alpha(x,t)$ is the Riesz–Feller fractional derivative of order α $(0 < \alpha \le 2)$ and skewness $\theta(|\theta| \le \min(\alpha, 2-\alpha), \theta \ne \pm 1)$ which is defined in terms of the Fourier transform in [1], i.e.

$$\mathcal{F}\left\{{}_xD_\theta^\alpha f(x);\omega\right\} = -\psi_\theta^\alpha(\omega)\hat{f}(\omega), \tag{1.3}$$

where $\psi_\theta^\alpha(\omega) = |\omega|^\alpha e^{i(\operatorname{sign}\omega)\theta\pi/2}$.

This problem is taken from [2], where the authors proposed a regularization scheme using convolution, and gave an *a priori* choice of parameter. The special case for order $\alpha = 2$ and skewness $\theta = 0$ in (1.1) i.e. the classical backward heat conduction problem has been considered by many authors, for example, [3–5]. Fractional calculus and fractional differential equations have been used recently to describe a range of problems in physics, chemistry, biology, mechanical engineering, signal processing and system identification, electricity, control theory, finance and fractional dynamics, refer to [6,7] etc.

Fractional differential equations with Riesz–Feller space-fractional derivative can be derived from the continuous-time random walk in statistical mechanics and have a wide range of applications in the theorem of probability distribution, especially modeling for the high frequency price dynamics in financial markets [8–10].

In general, the error estimate under an *a posteriori* parameter choice is hard to obtain, but it has practical applications, thus it is very valuable. There are many researchers considering the error estimate under an *a posteriori* parameter choice.

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Neubauer, in 1988, gave an *a posteriori* parameter choice strategy for the Tikhonov regularization in the presence of modeling error in [11]. In the same year, Engl and Gfrerer [12] extended the *a posteriori* parameter choice to general regularization methods for solving linear ill-posed problems. In 1999, a discrepancy-based *a posteriori* parameter choice strategy for the Tikhonov regularization of nonlinear ill-posed problems was considered in [13]. In 1999, Hämarik and Raus [14] thought about an *a posteriori* parameter choice using the iterated Tikhonov regularization method.

The most widely used method for the *a posteriori* parameter choice is the discrepancy principle, which we will also use in this paper. The development of the discrepancy principle has been explored widely. Some researchers used the discrepancy principle to solve different problems. George et al. in [15] used it to select the regularization parameter for the simplified regularization method. Blanchard and Mathé in [16] considered it for statistical inverse problems with application to conjugate gradient iteration. In [17], Engl proved that the Tikhonov regularization with the discrepancy principle leads to an optimal convergence rate. In [18], Nair et al. gave a unified conclusion under general source conditions and they provided an order optimal error bound in 2003. Anzengruber and Ramlau used it for Tikhonov-type functionals with nonlinear operators in 2010 in [19]. In this paper, in order to get a nice convergence rate, we use a new and generalized discrepancy principle to choose the regularization parameter, which is different from Morozov's discrepancy principle used in [18,20–23], and also distinct from variations of Morozov's discrepancy principle in [24–27]. Numerical examples show that our new discrepancy principle is effective.

The structure of the paper is as follows. In Section 2, we introduce the convolution regularization method. In Section 3, we propose an *a posteriori* parameter choice rule based on Morozov's discrepancy principle, and we obtain an error estimate for the convolution method. Numerical results are shown in Section 4. At last we give a conclusion in Section 5.

2. The convolution regularization method

In this section, we review the convolution regularization method given in [2].

For $g(x) \in L^2(\mathbb{R})$, denote $\hat{g}(\omega)$ as its Fourier transform, defined by

$$\hat{g}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{\mathbb{R}} e^{i\omega x} g(x) dx. \tag{2.1}$$

Let $\|\cdot\|$ denote the norm in $L^2(\mathbb{R})$. The Parseval formula is

$$\|g\| = \|\hat{g}\|. \tag{2.2}$$

Taking the Fourier transform to the space-fractional diffusion equation with respect to the variable $x \in \mathbb{R}$, we have

$$\mathcal{F}({}_xD_\theta^\alpha f(x)) = -\psi_\theta^\alpha(\omega)\hat{f}(\omega). \tag{2.3}$$

Consequently the problem (1.1)–(1.2) in frequency domain can be written as

$$\hat{u}_t(\omega, t) = -\psi_\theta^\alpha(\omega)\hat{u}(\omega, t), \tag{2.4}$$

$$\hat{u}(\omega, T) = \hat{f}(\omega). \tag{2.5}$$

The solution for problem (2.4)–(2.5) can be easily given by

$$\hat{u}(\omega, t) = e^{\psi_\theta^\alpha(\omega)(T-t)}\hat{f}(\omega). \tag{2.6}$$

It follows that,

$$\hat{u}(\omega, 0) = e^{\psi_\theta^\alpha(\omega)T}\hat{f}(\omega). \tag{2.7}$$

Firstly we rewrite problem (2.6) as the following linear equation:

$$\hat{K}\hat{u}(\omega, t) = \hat{f}, \tag{2.8}$$

where $\hat{K}\hat{u}(\omega, t) = e^{-\psi_\theta^\alpha(\omega)(T-t)}\hat{u}(\omega, t)$ with $\hat{K} = e^{-\psi_\theta^\alpha(\omega)(T-t)}$.

We assume that the measured data f^δ satisfies

$$\|f^\delta - f\| = \|\hat{f}^\delta - \hat{f}\| \le \delta, \tag{2.9}$$

where the constant $\delta > 0$ is called a noise level.

Then we define $L = (\hat{K}^*\hat{K})^{-\frac{T}{T-t}} = e^{2\,\mathrm{Re}\,\psi_\theta^\alpha(\omega)T}$, where $\hat{K}^*$ is the adjoint of $\hat{K}$.

Let us describe an *a priori* information for $u(x, t)$ in more detail. We introduce a Hilbert scale $(X_r)_{r\in R^+}$ according to $X_0 = L^2(\mathbb{R})$ and $X_r = D(L^{\frac{r}{2}}) \subseteq L^2(\mathbb{R})$ where

$$\|v\|_r \triangleq \left\| L^{\frac{r}{2}}\hat{v} \right\|, \quad r \in R^+, \tag{2.10}$$

is the norm in X_r (see [28]). The *a priori* smoothness condition for the unknown initial value $u(x, 0)$ is assumed

$$u(x, 0) \in M_{r,E} = \{v \in X_r, \|v\|_r \le E\}, \tag{2.11}$$

for some $0 \le r \le 1$.

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