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Boundary integral equation methods for the scattering problem by an unbounded sound soft rough surface with tapered wave incidence

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HIGHLIGHTS

- A rigorous derivation of BIE is presented for studying the scattering problem.
- Some properties of the integral equation in an energy space with weights are proved.
- The convergence of the numerical method is also obtained.

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ABSTRACT

In this paper, we consider the scattering problem of tapered acoustic wave by an unbounded sound soft surface. The scattering problem is modeled as a boundary value problem governed by the Helmholtz equation with Dirichlet boundary condition. Although the tapered wave is often introduced to realize asymptotic truncation for unbounded rough surface, the standard Helmholtz integral equations which derive for the scattering of plane waves by an arbitrary bounded obstacle are often used to generate benchmark numerical solutions. Different from the scattering of plane waves by an arbitrary bounded obstacle, we use the angular spectrum representation radiation condition to replace the Sommerfeld radiation condition, and derive a boundary integral equation for studying the scattering problem. Then we study the integral equation by the truncation method, whereby the integral equation posed on an unbounded region is approximated by an integral equation on a bounded region. Some properties of the integral equation in an energy space with weights are proved. Then the collocation method is used to solve the integral equation on a bounded region, and its convergence is also obtained.

1. Introduction

Acoustic and electromagnetic scattering from rough surfaces play a fundamental role in many research areas, such as geoscience, medical ultrasonic, applied optics, and remote sensing. Several review papers and textbooks are available (see e.g. [1–3]). For bounded rough surface, there are no fundamental mathematical questions arise in the formalism. For unbounded periodic surface, the surface integration can be collapsed to a single periodic cell (see e.g. [4–6]). For the general unbounded non-periodic rough surface, it is often to assume that the usual boundary integral equation formulations for

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problems of scattering by bounded obstacles remain valid when the boundary is unbounded. Despite this extensive practical interest, relatively little mathematical analysis of these problems has been carried out. Some appropriate radiation conditions are given as a part of the boundary value problem, since the usual Sommerfeld radiation condition is no longer valid due to the unboundedness of the rough surface (see e.g. [7]). In [8–10] J.A. DeSanto and P.A. martin derived the boundary integral equation for scattering by an unbounded rough surface with plane wave incidence. They used the angular spectrum representation radiation condition and proved the validity of the standard Helmholtz integral equations, but it is still not clear if the integral equation is well-posed, in particular whether it is uniquely solvable once a truncation has taken place. Chandler-Wilde S.N. and Zhang B. formulated a mathematically rigorous boundary value problem by using the upward-going radiation condition to modify the usual Sommerfeld radiation condition, and the "Dirichlet Green function" is used to replace the free space Green function for overcoming the lack of compactness of the solution functional space (see e.g. [11–13]). Recently, Chandler-Wilde S.N. and J. Elschner proved the well-posedness of a variational formulation in an energy space with weights [14]. Some other results have been obtained to establish well-posedness for three-dimensional rough surface scattering problems by integral equation methods (see Chandler-Wilde, Heinemeyer and Potthast [15,16], Thomas [17]) or variational formulations (see Chandler-Wilde, Monk and Thomas [18,19], Thomas [17]).

In fact, the boundary integral equation methods (BIEM) are widely used for practical computations of rough surface scattering problems. Particularly popular to calculate accurate solutions, capturing all the multiple reflections which can take place. It is known that the main difficulties of numerical calculation by BIEM for unbounded rough surface are the edge diffraction effects will occur when the rough surface is truncated to a finite computational region and efficient algorithm for the large-scale problem. For solving the proposed problems in the above, the asymptotic truncation techniques of boundary for BIEM are the main methods in the past decade. For example, Eric I. Thorsos [20,21], L. Tsang, J. Kong et al. [22] introduced tapered wave to realize the asymptotic truncation for rough surface, and they obtained the approximate solution by solving the standard Helmholtz integral equation. Y. Jin, Z. Li [23] used forward and backward method with spectrum acceleration algorithm to calculate bistatic scattering from fractal rough dielectric surface with tapered wave incidence. H. Ye, and Y. Jin [24] presented an empirical formula to select tapering parameter of the tapered incident wave for numerical simulation of electromagnetic scattering from rough surface. L.X. Guo, Y. Liang et al. [25] presented a high order integral small perturbation method (SPM) for the conducting rough surface scattering with the tapered wave incidence. In [26] the integral equation for tapered wave incidence is used. By estimating the double integral as a perturbed right-hand side of the integral equation to obtain the regularization parameter, the authors presented a regularized conjugate gradient method with fast multipole acceleration for computing the scattering problem from a one dimensional rough surface with the tapered wave incidence. However, they did not discuss the validity of the Helmholtz integral equation formulations for total field. Only J.A. DeSanto, P.A. Martin and E.I. Thorsos simply explained the validity of the integral equation for tapered wave incidence based on the results which they established for plane wave incidence, they treated the tapered wave as an approximation of the plane wave (see e.g. [9,20]). Extending the standard boundary integral equation method for the problems of scattering by bounded rough surface to the case when the surface is unbounded raises a number of questions. It is not clear that the standard boundary integral equation method still applies to derive the integral equation. Further, it is not clear if the integral equation is well-posed. This is even less clear once a truncation of the unbounded surface to a bounded surface has taken place.

So in this paper, we will derive the integral equation for tapered wave incidence and study the numerical method of the integral equation. Based on some estimations of the integrals over the large semicircle, we strictly derived the integral expression of the Helmholtz equation for the problem of 1D infinite rough surface scattering with Thorsos tapered incident wave by using Green's theory and the free space Green's function, then we studied the properties of the integral operators. J.A. DeSanto and P.A. Martin pointed out that the angular spectrum radiation condition is a crucial ingredient to establish the integral equation. But for the rough surface governed by a general function, we take a different approach to estimate the integrals over the large semicircle. This is the key result to establish the boundary integral equation. In Section 2, we present the scattering problem and some useful lemmas which is used in Section 3 to derive the integral expression and analyse the properties of the integral operators. In Section 4, we truncate the surface, and the truncated arc has length $2L$, and the collocation method is used to solve the integral equation. The existence of approximation solutions and the convergence of approximation method are showed. Finally, the error estimate of the approximate solution is given.

2. The boundary value problem and some preliminary lemmas

Consider the scattering of the Thorsos tapered wave [20] by an unbounded rough surface. The Thorsos tapered wave can be described by

$$u_{inc}(\bar{r}) = \exp\big(ik(x\sin\theta_{inc} - z\cos\theta_{inc})(1+w(\bar{r}))\big)\exp\left(\frac{-(x+z\tan\theta_{inc})^2}{g^2}\right), \tag{2.1}$$

where $w(\bar{r}) = \frac{1}{(kg\cos\theta_{inc})^2}\left(\frac{2(x+z\tan\theta_{inc})^2}{g^2} - 1\right)$, and satisfies nonhomogeneous Helmholtz equation

$$\Delta u_{inc} + k^2 u_{inc} = k^2 R, \quad \text{in } \Omega, \tag{2.2}$$