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Exponential convergence and Lagrange stability for impulsive Cohen–Grossberg neural networks with time-varying delays

Liangliang Li, Jigui Jian *

College of Science, China Three Gorges University, Yichang, Hubei, 443002, China

HIGHLIGHTS

- We discuss Lagrange stability for impulsive Cohen–Grossberg neural networks.
- We establish a new delay impulsive differential inequality.
- Some easily verified conditions of Lagrange exponential stability are obtained.
- Giving out the detail estimations of the exponential convergence ball.
- The results here generalize and improve the earlier publications.

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ABSTRACT

In this paper, the problem on exponential convergence and Lagrange exponential stability for a class of delayed Cohen–Grossberg neural networks with impulses effects is investigated. To this end, a new delay impulsive differential inequality is established, which improves and generalizes previously known criteria. By using the new inequality and coupling with the Lyapunov method, several sufficient conditions are derived to guarantee the global exponential stability in Lagrange sense and exponential convergence of the state variables of the discussed delayed Cohen–Grossberg neural networks with impulses effects. Meanwhile, the framework of the exponential convergence ball in the state space with a pre-specified convergence rate is also given. Here, the existence and uniqueness of the equilibrium points need not to be considered. Finally, some numerical examples with simulation show the effectiveness of the obtained results.

1. Introduction

In the past decade, Cohen–Grossberg neural networks have been extensively studied and developed due to its potential applications and its complexity [1,2]. Most widely studied and used neural networks can be classified as either continuous or discrete. But in practice applications, such as the state of electronic networks is often subject to instantaneous changes at certain instants, this is impulsive phenomena. Examples of impulsive phenomena can also be founded in other fields of automatic control system, artificial intelligence, robotics, etc. [3]. Impulsive neural network model belongs to new category of dynamical systems, which is neither continuous nor discrete ones.

As we know, neural network could be stabilized or destabilized by impulsive phenomena [4,5]. The presence of impulse means that the state trajectory does not preserve the basic properties. Thus, the theory of impulsive differential equations is

* Corresponding author. Tel.: +86 0717 6392370; fax: +86 0717 6392370.
E-mail addresses: lilianglianglll@126.com (L. Li), jianjigui@sohu.com, jiguijian@ctgu.edu.cn (J. Jian).

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quite necessary to be further investigated and has attracted the attention of many scientists. In [6], Guan et al. introduced impulse into Hopfield type neural networks with delay. Recently, the asymptotical or exponential stability of impulsive neural networks with delays has been widely studied, and many results on this topic have been proposed in the literature [7,5,8].

On the other hand, in a wide range of these applications, it is essential that the neural networks involved are multistable. Many research results on multistability of neural networks have been reported by [9–14], these dynamics maybe have multiple equilibrium and so many of them are unstable [11,12]. In [13,14], the authors considered the multistability for impulsive hybrid Hopfield neural networks and discrete-time Hopfield neural networks. Motivated by these concerns, it is worth mentioning that Lagrange stability [15] refers to the stability of the total system which does not require the information of equilibrium points. From the theoretical and application view, it is necessary to study the stability properties in Lagrange sense for impulsive Cohen–Grossberg neural networks.

To the best of our knowledge, few researchers study the stability in Lagrange sense for impulsive neural networks with time-varying delays. In [16], He and Wang applied nonlinear delay differential inequality to consider the attracting and invariant sets of impulsive delay Cohen–Grossberg neural networks. In [17], the authors proved a new invariance theorem for a class of special nonlinear impulsive dynamical systems. In this paper, we will establish a new delay impulsive differential inequality and apply inequality technique to discuss the stability in Lagrange sense and exponential convergence for impulsive Cohen–Grossberg neural networks with time-varying delays. Moreover, the convergence ball domain of delay impulsive neural networks are given. It is believed that the results are significant and useful for the applications and design of impulsive neural networks.

This paper is organized as follows. In Section 2, some assumptions, notations and lemmas are given. In Section 3, we establish a new delay impulsive differential inequality. Some criteria for Lagrange stability and exponential convergence are proposed in Section 4. In Section 5, two illustrative examples are given to illustrate the obtained results. Conclusions will be presented in Section 6.

2. Preliminaries

Based on the model in [6], we consider a class of Cohen–Grossberg neural networks with time-varying delays and impulses described by the following measure differential equations:

$$Dx_i(t) = -\alpha_i(x_i(t))\left[\beta_i(x_i(t)) - \sum_{j=1}^{n} a_{ij}f_j(x_j(t))DU_j - \sum_{j=1}^{n} b_{ij}f_j(x(t-\tau(t)))DW_j - I_i\right], \tag{1}$$

where $i = 1, 2, \ldots, n$, $n \geq 2$ is the number of neurons in the network, $x_i(t)$ denotes the state variable associated with the neuron, and $\alpha_i(x_i(t)) > 0$ represents an amplification function, $\beta_i(x_i(t))$ is an appropriately behaved function. $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are connection weights from neuron i to neuron j; I_i is a constant external input; $\tau(t)$ denotes transmission delay with $0 \leq \tau(t) \leq \tau = \sup_{s\geq 0}\{\tau(s)\}$. The operator D denotes the distributional derivative, bounded variation functions $U_j, W_j : [t_0, +\infty) \to \mathbb{R}$ are right-continuous on any compact subinterval of $[t_0, +\infty)$. Activation function $f_j(\cdot)$ shows us how the neurons respond to each other, which is integrable functions with respect to U_j and W_j, respectively. DU_j, DW_j represent the effect of sudden changes in the states of network (1) at the discontinuity points of U_j and W_j for $j = 1, 2, \ldots, n$. We assume that

$$DU_j = 1 + \sum_{k=1}^{+\infty} u_{jk}\delta(t-t_k), \qquad DW_j = 1 + \sum_{k=1}^{+\infty} w_{jk}\delta(t-t_k),$$

where $j = 1, 2, \ldots, n$. $k \in \mathbb{N} \triangleq \{1, 2, \ldots\}$, the fixed impulsive moments t_k satisfy $t_{k-1} < t_k$ and $\lim_{k\to+\infty} t_k = +\infty$, $\delta(t)$ is the Dirac impulsive function, which means that the state of network (1) has jumps at t_k for $k \in \mathbb{N}$. u_{jk} and w_{jk} represent the strength of impulsive effects of the jth neuron at time t_k and $t_k - \tau(t)$, respectively. If $u_{jk} = w_{jk} = 0$, then model (1) becomes continuous neural network models (see [2,18,19]).

Let $\mathbb{C}(X, Y)$ denotes the space of continuous mappings from the topological space X to the topological space Y. Especially, $\mathbb{C} \triangleq \mathbb{C}([t_0-\tau, t_0], \mathbb{R}^n)$ denotes the family of all the bounded variation and right-hand continuous $\mathbb{R}^n$-valued functions $\varphi(\cdot) = (\varphi_1(\cdot), \varphi_2(\cdot), \ldots, \varphi_n(\cdot))^T$ defined on any compact subinterval of $[t_0-\tau, t_0]$. Throughout the paper, we always assume that for any $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_n)^T \in \mathbb{C}$ neural networks (1) has at least one solution through (t_0, φ) denoted by $x(t, t_0, \varphi)$ (simply $x(t, \varphi)$ or $x(t)$ if no confusion should occur). For impulsive neural networks (1), its initial conditions are given by

$$x(t) = \varphi(t) \in \mathbb{C}. \tag{2}$$

For further discussion, some preliminaries are needed:

Assumption (H1). For each $i \in \{1, 2, \ldots, n\}$, the amplification function $\alpha_i(\cdot)$ is positive, bounded and satisfies

$$0 < \underline{\alpha}_i \leq \alpha_i(x_i(t)) \leq \overline{\alpha}_i < +\infty, \quad i = 1, 2, \ldots, n.$$

And let $\underline{\alpha} = \min\{\underline{\alpha}_1, \underline{\alpha}_2, \ldots, \underline{\alpha}_n\}$, $\overline{\alpha} = \max\{\overline{\alpha}_1, \overline{\alpha}_2, \cdots, \overline{\alpha}_n\}$.