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Design of joint between disk and shaft based on induction shrink fit

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ABSTRACT

The problem of fixing a disk on shaft by means of induction shrink fit is analyzed. The analysis consists of two principal parts. The first task is to propose an appropriate interference of the shaft and check the von Mises stress in the disk and shaft in acceptable operation regimes. The second task is to map the process of induction heating of the disk before its putting on the shaft (which represents a nonlinear and nonstationary coupled problem that is solved numerically, in the hard-coupled formulation). The methodology is illustrated with an example of the active wheel of a gas turbine whose results are discussed.

1. Introduction

Shrink fits represent firm joints of two metal parts that are based on elastic stress between them. Mostly they are used for transferring mechanical torques and can be found in numerous industrial or transport applications. The examples abound: shrunk-on rings, crankshafts, armature bandages in rotating machines, tires of railway wheels, etc. [1–4].

Shrink fits between disks and shafts are realized either by hydraulic pressing or by heating the disk, its setting on shaft and subsequent cooling of the whole system. In the latter case the disk can be heated by gas or inductively. During the process of heating the diameter of its internal bore grows to a value greater than the diameter of the shaft. Then the disk is put on the shaft and the whole system is cooled. The complete procedure is schematically shown in Fig. 1.

The shaft of external radius r_{A2} is manufactured with an interference Δr_{AB} with respect to the internal radius r_{B1} of the disk. After heating, the radius of the disk bore (along its whole axial length) must reach a value $r'_{\mathrm{B1}} > r_{\mathrm{A2}}$. Putting it on the shaft and cooling, the disk shrinks, which leads to a decrease of both radii r_{A2} and r'_{B1} to a common radius r_{C} that satisfies inequality $r_{\mathrm{B1}} < r_{\mathrm{C}} < r_{\mathrm{A2}}$ (in our considerations even the shaft is assumed elastic, i.e., its radius can also shrink). Shrinking of both parts produces at the place of contact a unit contact force $\pm f_{r0}$ (index 0 means "at rest") between them, see Fig. 1, which allows transferring the required mechanical torque.

To the knowledge of the authors, the paper represents the first study where the problem of an axisymmetric induction shrink fit is dealt with in its full complexity.

2. Procedure of solution

Consider a shrink fit consisting of a steel disk and shaft (Fig. 1) made of the same material. The fit must be able to transfer a prescribed mechanical torque $M_{\mathrm{mn_0 max}}$ at a given nominal revolutions n_0. The task is to propose the principal parameters of the fit and check them with respect to the required mechanical properties of the system.

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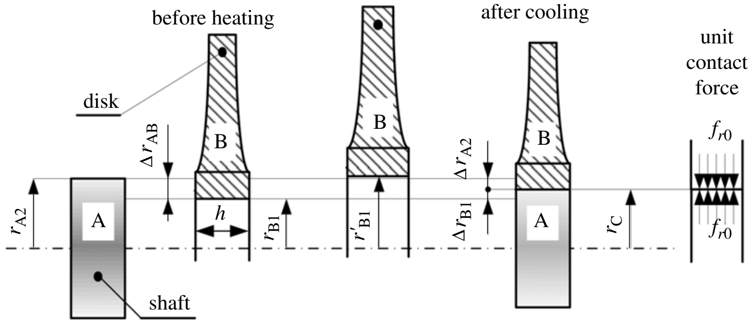

Fig. 1. Preparation of shrink fit.

The complete solution of the problem consists of the following partial steps:

- Starting from the geometry of the disk and torque $M_{\mathrm{m}n_0\mathrm{max}}$ to be transferred, first it is necessary to determine an appropriate interference Δr_{AB}. This is, however, a very complicated inverse problem. The simplest way of its solution is, therefore, to find the dependence of the distribution of the highest reduced stress in the disk on Δr_{AB}, choose an appropriate value Δr_{AB} for which the above stress is still well below the yield stress of the material and check the maximum transferable torque $M_{\mathrm{m}n\mathrm{max}}$ for revolutions $n \in \langle 0, n_0 \rangle$. The principal advantage of this way is that all the above dependences are almost linear.
- As for mechanical situation in the disk after its pressing on the shaft, its determination starts from knowledge of the unit force $\pm f_{rn}$ acting along their mutual contact for the whole range of revolutions $n \in \langle 0, n_0 \rangle$. This value then serves for computing the above reduced stress $\sigma_{\mathrm{red}n}$ (for example, $\sigma_{\mathrm{red}n\mathrm{Mi}}$ by the von Mises hypothesis) and its comparison with the yield stress σ_{y} of the steel used (there must hold $\sigma_{\mathrm{red}n\mathrm{Mi}} < \sigma_{\mathrm{y}}$). For growing revolutions, the effect of the centrifugal forces (acting mainly in the disk) may significantly influence the maximum acceptable reduced stress.
- Because of the centrifugal forces, the unit contact force $\pm f_{r0}$ at the place of the contact of both parts decreases with increasing revolutions n. For revolutions higher than n_0, the shrink fit has also to be checked with respect to the danger of slipping. This is closely connected with the release revolutions n_{rel} of the disk ($n_{\mathrm{rel}} > n_0$).
- Mapping of the process of induction heating of the disk. Its purpose is to find the parameters of the field current in the inductors (amplitude and frequency) securing that the internal radius of the disk reaches a value r'_{B1} in a reasonable time, still acceptable temperature and a good efficiency of the heating process.

3. Continuous mathematical model

The mathematical model of the task consists of two independent sub-models. The first of them is purely mechanical and its goal is to find the unit contact force $\pm f_{rn}$ acting along the contact place on the disk and shaft after their mutual pressing for the considered range of revolutions $n \in \langle 0, n_0 \rangle$, corresponding values of the maximum transferable torques $M_{\mathrm{m}n\mathrm{max}}$, and also von Mises stresses $\sigma_{\mathrm{red}n\mathrm{Mi}}$.

If these values are acceptable, another sub-model for the description of the process of induction heating is applied. This task represents a nonlinear and nonstationary triply coupled problem characterized by an interaction of magnetic field, temperature field and field of thermoelastic displacements. The most important physical properties of the material used are supposed to be functions of temperature.

As the arrangement of the system is axi-symmetric, the task will be described in co-ordinates r and z. The first (purely mechanical) sub-model is given by the isothermic Lamé equation in the form [5]

$$(\varphi + \psi)\mathrm{grad}(\mathrm{div}\,\boldsymbol{u}) + \psi\Delta\boldsymbol{u} + \boldsymbol{f}_{\mathrm{L}} = \boldsymbol{0},$$
$$\varphi = \frac{\nu E}{(1+\nu)(1-2\nu)}, \qquad \psi = \frac{E}{2(1+\nu)}, \tag{1}$$

where E denotes the Young modulus of elasticity of the material, ν is the Poisson coefficient of contraction, symbol $\boldsymbol{u} = (u_r, u_\theta, u_z)$ represents the vector of the displacements, and $\boldsymbol{f}_{\mathrm{L}} = (f_{\mathrm{L}r}, f_{\mathrm{L}z})$ stands for the vector of the volumetric forces. For the axisymmetric arrangement $u_\theta = 0$, and (neglecting the gravitational forces, whose influence may be considered insignificant) $f_{\mathrm{L}z} = 0$ as well. The component $f_{\mathrm{L}r}$ is given by the formula for the specific centrifugal force

$$f_{\mathrm{L}r} = \rho r \omega^2, \qquad \omega = 2\pi n/60, \tag{2}$$

where ρ denotes the specific mass of material of the disk.