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Nonlinear higher-order transient solver for magnetic fluid hyperthermia

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ABSTRACT

The article is devoted to numerical methods for transient solution of Pennes' bioheat equation as required for Magnetic Fluid Hyperthermia (MFH) modeling. Special attention has been paid to the role of non-linearity of blood perfusion and its influence on temperature distribution. The authors show that the higher-order time integration algorithms are highly advised for this type of problem, which should be classified as a stiff one. Popular low-order solvers give very different solutions. Furthermore, the application of adaptive time stepping scheme reduces calculation time and raises the efficiency of the simulation software.

1. Introduction

One of the most important aims of heat transfer modeling in living tissues is to be able to predict and control the level and the area of potential damage caused to tissues by extreme temperature. During, for example, hyperthermia treatment the damage should be limited to the cancer area, while the rest of the tissues should be left below the temperature of 45 °C. On the other hand, the modeling of heat transfer in soft tissues is also necessary for an accurate assessment of energy dissipation rate in joints, and for thermal analysis of the first stages of cryosurgical protocols (before freezing, when the effects of blood circulation are observed) [1]. In order to control the temperature during hyperthermia different types of thermometers are used, for instance: thermistors, thermocouples or infrared sensors [2]. The real measurements are more trustworthy, but numerical simulations are more flexible and often more convenient.

Magnetic fluid hyperthermia (MFH) is one of the most modern and extensively investigated temperature rise methods. In MFH the amount of energy provided to the body can be precisely controlled by the strength of magnetic field, the size of magnetic particles and their volume fraction in the tumor [3–7]. That is why, the investigation of bioheat-transfer phenomenon in living biological tissues requires the temporal and spatial estimation of temperature distribution.

Since Pennes' work [8], almost every model of temperature distribution in a human body is based on the bioheat transfer equation:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) - \rho_b c_b \omega (T - T_b) + Q_{met}, \tag{1}$$

where ρ is the tissue density, c—the tissue specific heat, ρ_b—the density of blood, c_b—the blood specific heat, κ—the tissue thermal conductivity, ω—the blood perfusion rate, T_b—the arterial blood temperature, Q_{met}—the metabolic heat source, and T is the local tissue temperature.

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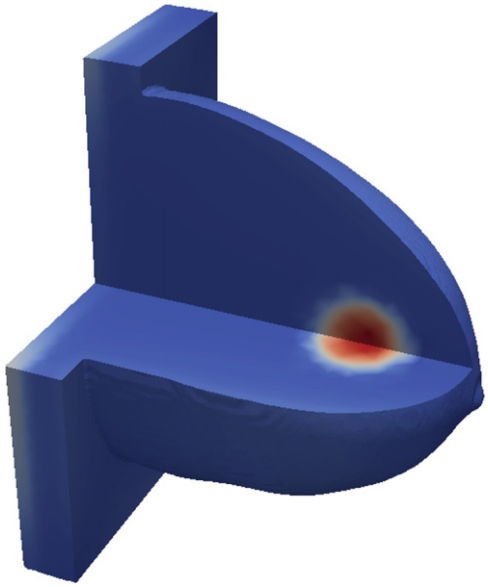

Fig. 1. Presented investigations are based on magnetic fluid hyperthermia for breast tumors.

Pennes' equation (1) describes the ability of a tissue to remove heat by both passive conduction and perfusion of the tissue by blood, in general. Pennes in his work assumed a constant-rate blood perfusion in the form of $\omega = V\rho_b$, where V and ρ_b are the perfusion rate per unit volume of tissue and the density of the blood, respectively. In this case, the temperature of venous blood is in equilibrium with the local temperature (T), and the arterial blood temperature (T_b) is constant. That means Pennes' original model describes blood perfusion with acceptable accuracy if there are no large vessels nearby, for example, liver [9]. However, the vascularized tissue often experiences increased perfusion as temperature increases and it is necessary to consider a more general form of (1) in which the blood perfusion ω is a function of temperature T (see for example [10–12]). Thus, the perfusion rate is the key parameter in calculating the heat transfer.

In this paper we have considered a more general form of Pennes' bioheat transfer formula, taking into account nonlinear blood mass flow, and external source of thermal energy provided to the body:

$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T) - c_b W (T - T_b) + Q_{met} + Q_{ext}, \tag{2}$$

where W is the mass flow rate of blood per unit volume of tissue in units of (kg/s/m^3), Q_{ext} is the external power density deposited in a tissue by the use of MFH.

An important factor of hyperthermia is also cooling associated with heat exchange with environment. It can be described by the convection boundary condition:

$$\kappa \frac{\partial T}{\partial n} = H(T_{amb} - T) \tag{3}$$

where H is the skin heat transfer coefficient and T_{amb} is the ambient temperature.

Among different technologies used for the increase of the body temperature, MFH is a special one, because it is based on superparamagnetic heat phenomenon. In this case superparamagnetic nanoparticles are injected into the cancerous tissue and then exposed to external magnetic field. The external low frequency magnetic field is passing through the body without interferences, only the area with nanoparticles is excited. The power density generated from nanoparticles can be understood as the internal source of heat.

In previous works [13], the authors have shown that the power losses from conductive heating based on eddy currents induced in the living tissues are negligible comparing with the power dissipated from nanoparticles. This means that the external power (Q_{ext}) in (2) is purely dependent on superparamagnetic heat phenomenon. In the current paper we are assuming that Q_{ext} is known from previous calculations, as presented in [13].

The choice of the discussed model was determined by our previous works connected with magnetic fluid breast cancer treatment (see Fig. 1). Though, now only two types of tissues are considered (breast fat and tumor), the presented investigation gives the general idea of the temperature distribution problem in low conducting lossy material.

The research is focused on the numerical methods, which can be applied to solve Pennes' equation. The main attention has been paid to the transient state solution and to the influence of tissue parameters, expressed as diffusion coefficient D and nonlinear perfusion function E (see Eq. (6)) on the solution. We have implemented the numerical time integration methods of different order showing their advantages and constraints when applied in real tissue coefficients of Pennes' equation with MFH excitation.