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Numerical solution of compressible and incompressible unsteady flows in channel inspired by vocal tract

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ABSTRACT

This study deals with the numerical solution of a 2D unsteady flow of a viscous fluid in a channel for low inlet airflow velocity. The unsteadiness of the flow is caused by a prescribed periodic motion of a part of the channel wall with large amplitudes, nearly closing the channel during oscillations. The channel is a simplified model of the glottal space in the human vocal tract. Four governing systems are considered to describe the unsteady laminar flow of a viscous fluid in the channel. The numerical solution is implemented using the finite volume method (FVM) and the predictor–corrector MacCormack scheme with artificial viscosity using a grid of quadrilateral cells. The unsteady grid of quadrilateral cells is considered in the form of conservation laws using the Arbitrary Lagrangian–Eulerian method. The numerical simulations of flow fields in the channel, acquired from a developed program, are presented for inlet velocity $\hat{u}_\infty = 4.12$ m s^{-1} and Reynolds number $Re_\infty = 4481$ and the wall motion frequency 100 Hz.

1. Introduction

A current challenging question is a mathematical and physical description of the mechanism for transforming the airflow energy in the glottis into the acoustic energy representing the voice source in humans. The voice source signal travels from the glottis to the mouth, exciting the acoustic supraglottal spaces, and becomes modified by acoustic resonance properties of the vocal tract [1].

Acoustic wave propagation in the vocal tract is usually modeled from incompressible flow models separately using linear acoustic perturbation theory, the wave equation for the potential flow [2] or the Lighthill approach on sound generated aerodynamically [3].

In reality, the airflow coming from the lungs causes self-oscillations of the vocal folds, and the glottis completely closes in normal phonation regimes, generating acoustic pressure fluctuations. In this study, the movement of the boundary channel is known, harmonically opening and nearly closing in the narrowest cross-section of the channel.

Goal is numerical simulation of flow in the channel which involves attributes of real flow causing acoustic perturbations.

2. Mathematical model

To describe the unsteady flow of a compressible viscous fluid in a channel, the 2D system of Navier–Stokes equations was considered as a mathematical model. The Navier–Stokes equations were transformed to non-dimensional form. The

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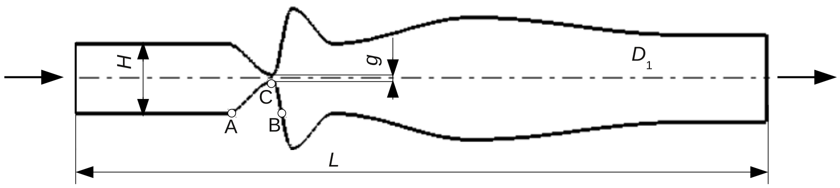

Fig. 1. The computational domain D. $L = 8$ (160 mm), $H = 0.8$ (16 mm), $g = 0.08$ (1.6 mm)—middle position.

reference variables for transformation are inflow variables (marked with the infinity subscript): the speed of sound $\hat{c}_\infty = 343\ \mathrm{m\,s^{-1}}$, density $\hat{\rho}_\infty = 1.225\ \mathrm{kg\,m^{-3}}$, dynamic viscosity $\hat{\eta}_\infty = 18\cdot 10^{-6}$ Pa s (for temperature $\hat{T}_\infty = 293.15$ K) and a reference length $\hat{L}_r = 0.02$ m. The system of Navier–Stokes equations is expressed in non-dimensional conservative form [4] as:

$$\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial \mathbf{R}}{\partial x} + \frac{\partial \mathbf{S}}{\partial y}\right). \tag{1}$$

W is the vector of conservative variables $\mathbf{W} = [\rho, \rho u, \rho v, e]^T$ where ρ denotes density, u and v are the components of the velocity vector and e is the total energy per unit volume. **F** and **G** are the vectors of inviscid fluxes and **R**, **S** are the vectors of viscous fluxes. The static pressure p in **F** and **G** is expressed by the state equation in the form

$$p = (\kappa - 1)\left[e - \frac{1}{2}\rho\left(u^2 + v^2\right)\right], \tag{2}$$

where $\kappa = 1.4$ is the ratio of specific heats.

General Reynolds number in (1) is computed from reference variables $\mathrm{Re} = \hat{\rho}_\infty \hat{c}_\infty \hat{L}_r / \hat{\eta}_\infty$. The non-dimensional dynamic viscosity in the dissipative terms is a function of temperature in the form $\eta = (T/T_\infty)^{3/4}$.

The system of equations (1) and (2) is the so-called *Full system*. Three other governing systems based on the Navier–Stokes equations presented, depend on the expression of state equation for static pressure p which depends on energy flow condition in the system. *Iso-energetic system* is a 2D system (1) where $\mathbf{W} = [\rho, \rho u, \rho v]^T$. The system is closed with pressure expression which is independent on the total energy variable e

$$p = \frac{\rho}{\kappa}\left[1 + \frac{\kappa - 1}{2}\left(\frac{\hat{u}_\infty}{\hat{c}_\infty}\right)^2 - \frac{\kappa - 1}{2}(u^2 + v^2)\right]. \tag{3}$$

Adiabatic system is a 2D system (1) where $\mathbf{W} = [\rho, \rho u, \rho v]^T$ and is closed with pressure expression dependent only on density

$$p = \frac{1}{\kappa}\rho^\kappa. \tag{4}$$

Incompressible system—Because $\rho = const$ for numerical solution of the system Artificial Compressibility Method [5] is used. The system of Navier–Stokes equations goes to form

$$\Gamma\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = \frac{1}{\mathrm{Re}}\left(\frac{\partial \mathbf{R}}{\partial x} + \frac{\partial \mathbf{S}}{\partial y}\right), \tag{5}$$

where $\mathbf{W} = [p/\rho, u, v]^T$, $\Gamma = [1/\beta^2, 1, 1]^T$, $\beta^2 > 1$. The system (5) describes steady state flow of incompressible viscous fluid.

3. Computational domain and boundary conditions

The bounded computational domain D used for the numerical solution of flow field in the channel is shown in Fig. 1. The domain is a symmetric channel, the shape of which is inspired by the shape of the trachea (inlet part), vocal folds, false vocal folds and supraglottal spaces (outlet part) in the human vocal tract. The upper and the lower boundaries are the channel walls. A part of the walls changes its shape between the points A and B according to given harmonic function of time and axial coordinate. The gap width is the narrowest part of the channel (in point C) and is oscillating between the minimum $g_{\min} = 0.4$ mm and maximum $g_{\max} = 2.8$ mm.

The boundary conditions are considered in the following formulation:

1. Upstream conditions: $u_\infty = \frac{\hat{u}_\infty}{\hat{c}_\infty}$; $v_\infty = 0$; $\rho_\infty = 1$; p_∞ is extrapolated from D.
2. Downstream conditions: $p_2 = 1/\kappa$; $(\rho, \rho u, \rho v)$ are extrapolated from D.