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Finite element simulation of three-dimensional particulate flows using mixture models

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ABSTRACT

In this paper, we discuss the numerical treatment of three-dimensional mixture models for (semi-)dilute and concentrated suspensions of particles in incompressible fluids. The generalized Navier–Stokes system and the continuity equation for the volume fraction of the disperse phase are discretized using an implicit high-resolution finite element scheme, and maximum principles are enforced using algebraic flux correction. To prevent the volume fractions from exceeding the maximum packing limit, a conservative overshoot limiter is applied to the converged convective fluxes at the end of each time step. A numerical study of the proposed approach is performed for 3D particulate flows over a backward-facing step and in a lid-driven cavity.

1. Introduction

Flows of incompressible fluids carrying suspensions of rigid particles occur very commonly in science, nature, and technology. Due to the complexity of mechanisms that govern fluid–particle and particle–particle interactions, numerical simulation of such flows belongs to the most challenging problems in Computational Fluid Dynamics (CFD). The heterogeneous nature of disperse two-phase flows has engendered a hierarchy of models that cover the whole range of relevant scales and differ greatly in their complexity.

In this paper, we consider averaged continuum models in which the effective density and viscosity of the mixture depend on the local volume fraction of the disperse phase [1,2]. In the dilute regime, we use an analog of the Boussinesq approximation for natural convection flows. The numerical implementation of the presented mixture model is based on the methodology we developed in [3] for buoyancy-driven turbulent bubbly flows.

When it comes to simulating dense suspensions, it is essential to ensure that the volume fraction of the disperse phase is bounded above. A typical model for dense suspensions incorporates an interparticle stress term designed to keep the particle volume fraction below the close-packing value [4–6]. Leiderman and Fogelson [7] multiplied the convective flux by a monotonically decreasing function of the volume fraction to impair the ability of particles to move into regions packed with other particles.

The flux-corrected transport (FCT) algorithm proposed in [8] combines the idea of Leiderman and Fogelson [7] with algebraic flux correction [9]. Instead of modifying the convective flux at the continuous level, we decompose the discretized convective term into numerical fluxes and limit the magnitude of these fluxes so as to get rid of unrealistic maxima. The advantages of constraining the discrete solution in this way are twofold. First, there is no need for tuning any free parameters

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or choosing the 'right' damping function for the convective flux. Second, the employed limiting strategy does not prevent the particles from leaving the regions of maximum concentration.

In the original publication [8], we applied the overshoot limiter to a 2D implosion problem with a prescribed velocity field. In the present paper, we use the same strategy to enforce the maximum principle for volume fractions in 3D mixture models of particulate flows. The numerical results for two test problems (backward-facing step and lid-driven cavity) illustrate the ability of the proposed scheme to handle dilute and concentrated suspensions.

2. Mixture model

In mixture models of disperse two-phase flows, the velocity $\mathbf{u}$ and pressure p of the suspension are given by the incompressible Navier–Stokes equations

$$\frac{\partial(\rho\mathbf{u})}{\partial t} + \nabla\cdot(\rho\mathbf{u}\otimes\mathbf{u}) = -\nabla p + \nabla\cdot(2\mu\mathcal{D}(\mathbf{u})) + \rho\mathbf{g}, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0, \tag{2}$$

where ρ is the effective density, $\mathcal{D}(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^T)$ is the strain rate tensor, μ is the effective viscosity, and $\mathbf{g}$ is the gravitational acceleration.

The hydrodynamic behavior of the mixture depends on the local volume fraction α of the disperse phase. In the fully Eulerian modeling framework, the evolution of α is governed by the hyperbolic continuity equation

$$\frac{\partial\alpha}{\partial t} + \nabla\cdot(\alpha\mathbf{u}_p) = 0, \tag{3}$$

where $\mathbf{u}_p$ is the average velocity of the particles. The average velocity of the fluid phase is denoted by $\mathbf{u}_f$. The relative velocity $\mathbf{u}_r = \mathbf{u}_p - \mathbf{u}_f$ is known as the slip velocity, settling velocity, or sedimentation velocity. It can be determined using empirical correlations (see below).

The effective density and momentum of the mixture are given by [10]

$$\rho = (1-\alpha)\rho_f + \alpha\rho_p, \tag{4}$$

$$\rho\mathbf{u} = (1-\alpha)\rho_f\mathbf{u}_f + \alpha\rho_p\mathbf{u}_p, \tag{5}$$

where ρ_p is the density of the solid and ρ_f is the density of the fluid. It follows that $\mathbf{u}_p$ can be expressed in terms of $\mathbf{u}$ and $\mathbf{u}_r$ as follows [1]:

$$\mathbf{u}_p = \mathbf{u} + \frac{1-\alpha}{1+\alpha\Theta}\mathbf{u}_r, \quad \Theta = \frac{\rho_p}{\rho_f} - 1.$$

The model is closed by problem-dependent constitutive laws for $\mathbf{u}_r$ and μ.

3. Boussinesq approximation

In the dilute flow regime, the mixture behaves as a weakly compressible fluid and can be modeled using an analogy to the Boussinesq approximation for natural convection flows. The use of this approach in the context of disperse two-phase flow modeling goes back to the work of Lapin and Lübbert [11] and Sokolichin et al. [12,13]. As shown by Lalli [2], it is well suited for simulating dilute suspensions of particles in incompressible fluids.

Using $\rho \approx \rho_f$ in the left-hand side of the momentum equation (1) and the constant effective viscosity $\mu \approx \mu_f$ in the right-hand side, one obtains

$$\rho_f\left[\frac{\partial u}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u}\right] = -\nabla p + \mu_f\Delta\mathbf{u} + \rho_f\mathbf{g} + \alpha(\rho_p - \rho_f)\mathbf{g}.$$

Division by the constant density ρ_f yields the Boussinesq-like model [2]

$$\frac{\partial u}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{u} = -\nabla\tilde{p} + \nu_f\Delta\mathbf{u} + \alpha\Theta\mathbf{g}, \tag{6}$$

$$\nabla\cdot\mathbf{u} = 0. \tag{7}$$

The kinematic viscosity ν_f and modified pressure $\tilde{p}$ are defined by

$$\nu_f = \frac{\mu_f}{\rho_f}, \qquad \nabla\tilde{p} = \frac{1}{\rho_f}\nabla p - \mathbf{g}.$$