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Optimality and duality for minimax fractional programming with support function under (C, α, ρ, d)-convexity

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ABSTRACT

Sufficient optimality conditions are established for a class of nondifferentiable generalized minimax fractional programming problem with support functions. Further, two dual models are considered and weak, strong and strict converse duality theorems are established under the assumptions of (C, α, ρ, d)-convexity. Results presented in this paper, generalizes several results from literature to more general model of the problems as well as for more general class of generalized convexity.

1. Introduction

Chinchuluun and Pardalos [1] considered optimality conditions and duality for multiobjective programming problems, multiobjective fractional programming problems and multiobjective variational programming problems under the assumptions of (C, α, ρ, d)-convexity. Liang et al. [2] introduced a unified formulation of the generalized convexity and obtained some results on optimality conditions and duality theorems for a single objective programming problem. Yuan et al. [3] studied nondifferentiable minimax fractional programming problem for locally Lipschitz functions under the assumptions of (C, α, ρ, d)-convexity. Yuan et al. [4] considered (C, α, ρ, d)-type-I functions and presented sufficient optimality conditions and duality results for a nondifferentiable multiobjective programming problem for Lipschitz functions. Chinchuluun et al. [5] extended the results of [4] to multiobjective fractional case.

Long et al. [6] studied a class of nondifferentiable multiobjective fractional programs in which every component of the objective function contains a term involving the support function of a compact convex set and obtained Kuhn–Tucker necessary and sufficient optimality conditions, duality and saddle point results for weakly efficient solutions of the nondifferentiable multiobjective fractional programming problems. Recently, Long [7] considered a class of nondifferentiable multiobjective fractional programming problem in which the numerator of every component of the objective function contains a term involving the support function of a compact convex set. Long [7] established sufficient optimality conditions and duality results for the problem involving (C, α, ρ, d)-convexity. Kim and Kim [8] established sufficient optimality conditions and duality results for nondifferentiable generalized fractional programming problem in which the numerator as well as the denominator of every component of the objective function contains a term involving the support function of compact convex sets. Kim and Kim [8] obtained these results under the (V, ρ)-Invexity assumptions.

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In this paper, we have considered a class of nondifferentiable multiobjective fractional programming problem in which the numerator as well as denominator of every component of the objective function contains a term involving the support functions of convex sets. We have obtained sufficient optimality conditions and duality results for the problem under the assumption of (C, α, ρ, d)-convexity.

2. Notations and preliminaries

We consider the following nondifferentiable generalized minimax fractional programming problem with support function (GMFPS):

$$\text{(GMFPS)} \min_{x\in R^n} \sup_{y\in Y} \frac{F(x,y)}{G(x,y)} = \min_{x\in R^n} \sup_{y\in Y} \frac{f(x,y)+s(x|C)}{g(x,y)-s(x|D)}$$

$$\text{Subject to } h_j(x) + s\left(x|E_j\right) \le 0, \quad j = 1,\dots,p, \tag{1}$$

where Y is a compact subset of $R^m, f, g : R^n \times R^m \to R$, and $h_j : R^n \to R^p$ $(j = 1,\dots,p)$, are continuously differentiable functions on $R^n \times R^m$. C, D and E_j $(j = 1,\dots,p)$ are compact convex sets in R^m, and $s(x|C)$, $s(x|D)$ and $s\left(x|E_j\right)$, $(j = 1,\dots,p)$ designate the support functions of compact sets and $F(x,y) \ge 0$ and $G(x,y) > 0$ for all feasible x. Let R^n be the n-dimensional Euclidean space and R^n_+ be nonnegative orthant of R^n. Let X be an open subset of R^n. Assume that $\alpha : X \times X \to R_+ \setminus \{0\}$, $\rho \in R$ and $d : X \times X \to R_+$ satisfies $d(x,x_0) = 0 \Leftrightarrow x = x_0$. Let $C : X \times X \times R^n \to R$ be a function which satisfies $C_{(x,x_0)}(0) = 0$ for any $(x,x_0) \in R^n \times R^m$.

Let $S = \{x \in X : g(x) \le 0\}$ denote the set of all feasible solutions of (GMFPS). For each $(x,y) \in R^n \times R^m$, we define $\phi(x,y) = \frac{f(x,y)+s(x|C)}{g(x,y)-s(x|D)}$, such that for each $(x,y) \in S \times Y$, $f(x,y) + s(x|C) \ge 0$ and $g(x,y) - s(x|D) > 0$.

Let us define the following sets for every $x \in S$:

$$J(x) = \left\{j \in J | h_j(x) + s\left(x|E_j\right) = 0\right\},$$

$$Y(x) = \left\{y \in Y \left| \frac{f(x,y)+s(x|C)}{g(x,y)-s(x|D)} = \sup_{z\in Y} \frac{f(x,z)+s(x|C)}{g(x,z)-s(x|D)} \right.\right\}.$$

$$K(x) = \left\{\begin{array}{l}\left(s,t,\tilde{y}\right) \in N \times R^s_+ \times R^m : 1 \le s \le n+1, t = (t_1,\dots,t_s) \in R^s_+ \\ \quad \text{with} \sum_{i=1}^{s} t_i = 1, \tilde{y} = (\bar{y}_1,\dots,\bar{y}_s) \text{ and } \bar{y}_i \in Y(x), i = 1,2,\dots,s\end{array}\right\}.$$

Since f and g are continuously differentiable and Y is compact subset of R^m, it follows that for each $x^* \in S$, $Y(x^*) \neq \phi$. Thus for any $\bar{y}_i \in Y(x^*)$, we have a positive constant

$$k_0 = \phi\left(x^*, \bar{y}_i\right) = \frac{f(x^*,\bar{y}_i)+s(x^*|C)}{g(x^*,\bar{y}_i)-s(x^*|D)}.$$

The problem (GMFPS) is more general than that of the problems considered by Kim and Kim [8] as well as by Long [7].

Definition 2.1. A function $C : X \times X \times R^n \to R$ is said to be convex on R^n iff for and fixed $(x,x_0) \in X \times X$ and for any $y_1, y_2 \in R^n$, one has

$$C_{(x,x_0)}(\lambda y_1 + (1-\lambda) y_2) \le \lambda C_{(x,x_0)}(y_1) + (1-\lambda) C_{(x,x_0)}(y_2) \quad \text{for all } \lambda \in]0,1[.$$

Definition 2.2 (*[9]*)**.** A differentiable function $h : X \to R$ is said to be (C, α, ρ, d)-convex at $x_0 \in X$ iff for any $x \in X$,

$$\frac{h(x)-h(x_0)}{\alpha(x,x_0)} \ge C_{(x,x_0)}(\nabla h(x_0)) + \rho \frac{d(x,x_0)}{\alpha(x,x_0)}.$$

The function h is said to be (C, α, ρ, d)-convex on X iff it is (C, α, ρ, d)-convex at every point in X. In particular, h is said to be strongly (C, α, ρ, d)-convex on X iff $\rho > 0$.

Remark 2.1. If the function C is sublinear with respect to the third argument, then the (C, α, ρ, d)-convexity is the same as the (F, α, ρ, d)-convexity introduced by Liang et al. [10].

Let K be a compact convex set in R^n. The support function of K is denoted by $s(x|K)$ and defined by $s(x|K) := \max\left\{x^t y : y \in K\right\}$.