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A dynamic viscoelastic contact problem with normal compliance

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ABSTRACT

A dynamic contact problem between a viscoelastic body and a deformable obstacle is numerically considered in this work. The contact is modeled by using the well-known normal compliance contact condition. The variational formulation of this problem is written in terms of the velocity field and it leads to a parabolic nonlinear variational equation. An existence and uniqueness result is stated. Fully discrete approximations are then introduced by using the finite element method to approximate the spatial variable, and a hybrid combination of the implicit and explicit Euler schemes to discretize the time derivatives. An a priori error analysis is recalled. Then, an a posteriori error analysis is provided extending some results already obtained in the study of the heat equation, other parabolic equations and the quasistatic case. Upper and lower bounds are proved. Finally, some two-dimensional numerical simulations are presented to demonstrate the accuracy and the behavior of the error estimators.

1. Introduction

In this paper, a dynamic contact problem involving a viscoelastic body and a deformable obstacle is considered from the numerical point of view. Different problems dealing with this kind of viscoelastic materials have been studied in the past thirty years, and they are interesting because many metals or crystals can be modeled by using this theory.

Since the first results provided in [1], a large number of papers dealing with dynamic viscoelastic contact problems have been published including issues such as the existence and uniqueness of solutions and their properties (see, for instance, [2–14]) or their numerical analysis (see, e.g., [15–22]).

In this paper, we revisit a well-known dynamic contact problem involving a linear viscoelastic body. An a priori analysis is recalled (to our knowledge, it was not published yet), by using some ideas employed in [23] for the case including the mechanical damage. Then, an a posteriori error analysis is provided extending some arguments already applied in the study of the heat equation (see, e.g., [24,25]), some parabolic equations (see [26]), the Stokes equation (see [27]) or the recently considered quasistatic case (see [28]). As far as we know, this is the first time when such a posteriori error techniques are applied to the study of dynamic contact problems in solid mechanics, and it continues [29], where the contact with a deformable obstacle was not considered.

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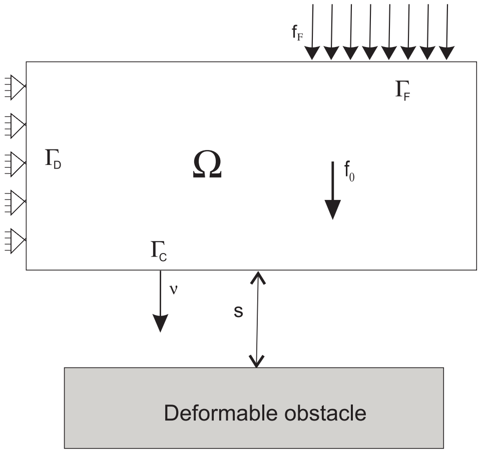

Fig. 1. A viscoelastic body in dynamic contact with a deformable obstacle.

The paper is structured as follows. In Section 2, the mechanical model and its variational formulation are described following the notation and assumptions introduced in [30,10]. Then, a fully discrete scheme is introduced in Section 3, by using the finite element method to approximate the spatial variable and a hybrid combination of the implicit and explicit Euler schemes to discretize the time derivatives. An a priori error estimates result, proved proceeding as in the case of including the mechanical damage, is recalled. Then, extending some results obtained in the study of quasistatic viscoelastic problems and the heat equation, an a posteriori error analysis is shown in Section 4, providing an upper bound for the error, Theorem 5, and a lower bound, Theorem 6. Finally, some numerical simulations, involving two-dimensional examples, are presented in Section 5.

2. Mechanical problem and its variational formulation

In this section, we present a brief description of the model (details can be found in [30,10]).

Denote by $\mathbb{S}^d$ the space of second order symmetric tensors on $\mathbb{R}^d$ and by "$\cdot$" and $\|\cdot\|$ the inner product and the Euclidean norms on $\mathbb{R}^d$ and $\mathbb{S}^d$.

Let $\Omega \subset \mathbb{R}^d$, $d = 2, 3$, denote a domain occupied by a viscoelastic body with a Lipschitz boundary $\Gamma = \partial\Omega$ decomposed into three disjoint parts Γ_D, Γ_F and Γ_C such that meas $(\Gamma_D) > 0$ and meas $(\Gamma_C) > 0$. Let $[0, T]$, $T > 0$, be the time interval of interest. The body is being acted upon by a volume force with density $\boldsymbol{f}_0$, it is clamped on Γ_D and surface tractions with density $\boldsymbol{f}_F$ act on Γ_F. Finally, we assume that the body may come in contact with a deformable obstacle on the boundary part Γ_C, which is located at a distance s, measured along the outward unit normal vector $\boldsymbol{\nu} = (\nu_i)_{i=1}^d$ (see Fig. 1).

Let $\boldsymbol{x} \in \Omega$ and $t \in [0, T]$ be the spatial and time variables, respectively. In order to simplify the writing, we do not indicate the dependence of the functions on $\boldsymbol{x}$ and t. Moreover, a dot above a variable represents its derivative with respect to the time variable.

Let $\boldsymbol{u} = (u_i)_{i=1}^d \in \mathbb{R}^d$, $\boldsymbol{\sigma} = (\sigma_{ij})_{i,j=1}^d \in \mathbb{S}^d$ and $\boldsymbol{\varepsilon}(\boldsymbol{u}) = (\varepsilon_{ij}(\boldsymbol{u}))_{i,j=1}^d \in \mathbb{S}^d$ denote the displacement field, the stress tensor and the linearized strain tensor, respectively. We recall that

$$\varepsilon_{ij}(\boldsymbol{u}) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right), \quad i, j = 1, \ldots, d.$$

The body is assumed viscoelastic and it satisfies the following constitutive law (see, for instance, [1]),

$$\boldsymbol{\sigma} = \mathcal{A}\boldsymbol{\varepsilon}(\dot{\boldsymbol{u}}) + \mathcal{B}\boldsymbol{\varepsilon}(\boldsymbol{u}), \tag{1}$$

where $\mathcal{A} = (a_{ijkl})$ and $\mathcal{B} = (b_{ijkl})$ are the fourth-order viscous and elastic tensors, respectively.

We turn now to describe the boundary conditions.

On the boundary part Γ_D we assume that the body is clamped and thus the displacement field vanishes there (and so $\boldsymbol{u} = \boldsymbol{0}$ on $\Gamma_D \times (0, T)$). Moreover, since the density of traction forces $\boldsymbol{f}_F$ is applied on the boundary part Γ_F, it follows that $\boldsymbol{\sigma}\boldsymbol{\nu} = \boldsymbol{f}_F$ on $\Gamma_F \times (0, T)$.

Finally, the contact is assumed with a deformable obstacle and so, the well-known normal compliance contact condition is employed for its modeling (see [31,12]); that is, the normal stress $\sigma_\nu = \boldsymbol{\sigma}\boldsymbol{\nu} \cdot \boldsymbol{\nu}$ on Γ_C is given by

$$-\sigma_\nu = p(u_\nu - s),$$

where $u_\nu = \boldsymbol{u} \cdot \boldsymbol{\nu}$ denotes the normal displacement in such a way that, when $u_\nu > s$, the difference $u_\nu - s$ represents the interpenetration of the body's asperities into those of the obstacle. The normal compliance function p is prescribed and it satisfies $p(r) = 0$ for $r \leq 0$, since then there is no contact. As an example, one may consider

$$p(r) = c_p\, r_+,$$