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Pricing and hedging of long dated variance swaps under a 3/2 volatility model

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ABSTRACT

This paper investigates the pricing and hedging of variance swaps under a 3/2 volatility model using explicit formulae. Pricing and hedging is performed under the benchmark approach, which only requires the existence of the numéraire portfolio. The growth optimal portfolio is used as numéraire together with the real world probability measure as pricing measure. This real world pricing concept provides minimal prices for variance swaps even when an equivalent risk neutral probability measure does not exist.

1. Introduction

A significant source of risk in a financial market is the uncertainty of the volatility of equity indices. During financial turmoil, volatility risk is of extreme importance to investors and derivative traders. Additionally, due to large and frequent shifts in the volatility of various assets in volatile periods, there is a growing practical need for appropriate models that allow for the realistic modeling of volatility, the pricing of related financial instruments and the hedging of volatility risk. In 1993, the Chicago Board Options Exchange (CBOE) introduced a volatility index, the VIX, see Fig. 1.1, based on implied volatilities of options on the S&P 500 index, see Fig. 1.2. The volatility of this diversified index has attracted the most attention in the almost of literature on volatility derivatives. As such, this paper considers as security of interest a similarly diversified equity index and studies its volatility and related derivatives. The S&P 500 with the VIX are employed in examples.

Variance swaps have been traded in over-the-counter markets since the collapse of Long Term Capital Management in late 1998. In particular, variance swaps on stock indices have been traded actively as a hedge for volatility risk. Investors and fund managers alike have developed an interest in volatility derivatives since these instruments may substantially increase the value of their holdings, even if the equity market index experiences a major drawdown. By the well-known leverage effect for equity indices, at least theoretically, volatility derivatives can provide some protection against severe market downturns. How effective such portfolio insurance is, from a macro-economic view point, remains an open question. In particular, when a large and increasing number of pension funds, insurance companies and other investors rely on this type

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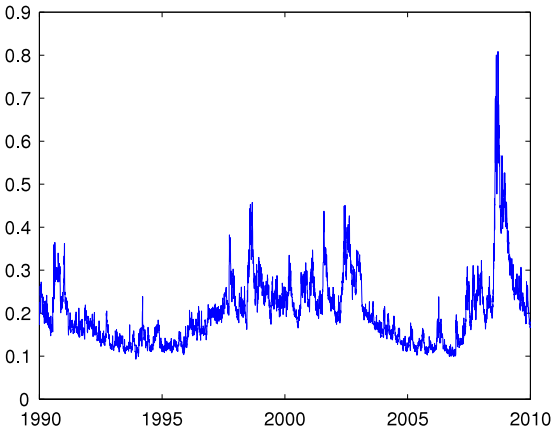

Fig. 1.1. VIX from 1990 to 2010.

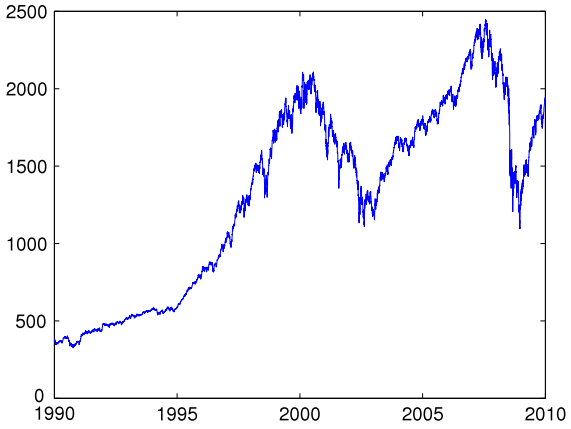

Fig. 1.2. S&P 500 from 1990 to 2010.

of insurance, it is not clear whether the sellers of variance swaps will be able to serve their obligations in a severe market downturn.

Fears about sovereign debt sustainability are valid explanations for the downturn in equity markets as experienced in May 2006, May 2010 and August 2011. However, they do not fully explain the extreme magnitude of the shocks observed and the repeated occurrence in European and US markets close. Hedging activity for long-dated derivatives by large players might exacerbate the occurrence of such volatility spikes. On the other hand, prolonged periods of calm markets may encourage hedge funds and other players to bet that equity market volatility would remain low, selling volatility to banks through variance swaps. The banks, then hedge against the variance swaps in two ways: they sell vanilla options to hedge their volatility exposure and delta hedge these vanilla options using cash equities. Consequently, they trade in the direction of the day's trend, thus exacerbating the decline in market and rise in volatility. This is illustrative of pro-cyclical feedback hedging effects, see also [1]. As variance swaps are priced off the market's closing price, these feedback effects are concentrated at the end of the day, implying dry in market liquidity. This is just the effect from hedging variance swaps on a macro economic scale. It is clear by the leverage effect that volatility itself will spike when the market experiences a severe downturn. It is possible that the implemented hedging strategies will not find any liquidity in the direction they want to trade. As a consequence, the players who issued variance swaps have no longer any protection. We will argue in this paper that the market is coming closer and closer to a potential market failure with the need of bailouts if the already macro economic scale of issued variance swaps continuous to increase. It is then not a question whether there will be a crisis it is only a question of time that a crisis occurs that is caused by variance swap hedging.

There exists a substantial literature on volatility modeling, and one may refer to Cont and Tankov [2] as one of many references. Several papers, which explore stochastic volatility models, have pointed at the seemingly undesirable property of some models, where the moments of squared volatility of higher order than one may become infinite in finite time. Examples are given in [3–5]. Furthermore, there exist various papers discussing the general problem of pricing and hedging variance swaps, including Brenner et al. [6], Grünbuchler and Longstaff [7], Carr and Madan [8], Chriss and Morokoff (1999), Demeterfi et al. [9], Brockhaus and Long [10], Matytsin [11], Javaheri et al. [12], Swishchuk [13], Howison et al. [14], Carr