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Using the linear sampling method and an improved maximum product criterion for the solution of the electromagnetic inverse medium problem

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ABSTRACT

We present a Tikhonov parameter choice approach for three-dimensional reconstructions based on a maximum product criterion (MPC) which provides a regularization parameter located in the concave part of the L-curve in log–log scale. Our method, baptized Improved Maximum Product Criterion (IMPC), is an extension of the MPC method developed by Bazán et al. for two-dimensional reconstructions. In the 3D framework, IMPC computes the regularization parameter via a fast iterative algorithm and requires no *a priori* knowledge of the noise level in the data. It is applied on the linear sampling method for solving the electromagnetic inverse medium problem in the 3D framework. The effectiveness of IMPC is illustrated with numerical examples involving more than one scatterer.

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1. Introduction

In this work we shall consider the scattering of a time harmonic electromagnetic wave with frequency in the resonance region by a finite number of three dimensional scatterers each of which is a penetrable isotropic medium. The inverse scattering problem we are considering is related to the determination of the shape of a penetrable scatterer in $\mathbb{R}^3$.

The approach we shall use to solve the inverse electromagnetic scattering problem is a combination of the well known linear sampling method originally developed in the acoustic context by Colton and Kirsch [1] and an improved version of the maximum product criterion (MPC) developed by Bazán et al. [2] for 2D reconstructions. It is widely known that the linear sampling method does not require *a priori* information about either the boundary condition or the connectivity of the scatterer, however it does require multistatic data at a single frequency. Due to the ill-posedness of the inverse problem, the linear sampling method yields an ill-conditioned system of linear equations whose solution requires a regularization method in order to handle correctly the presence of noise in the data. In particular, this solution requires the use of Tikhonov regularization method equipped with Morozov's generalized discrepancy principle as parameter choice rule [3,1,4–8], which generally involves the computation of the zeros of the discrepancy function at each point of the grid. In addition, the noise level in the data should be known *a priori*, something that in real life applications is not the case in general.

For electromagnetism, the linear sampling method has already been analyzed for perfect conducting scatters [9], imperfect conductors with impedance boundary data [7] and penetrable scatterers [10]. In particular, as presented in [10],

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the mathematical justification of the method is based on the formulation of an interior transmission problem for which a weak solution is shown to exist. Then the theoretical justification follows by extending to electromagnetics methods used for the acoustic problem.

The Maximum Product Criterion (MPC), originally developed for two dimensional reconstructions [2], employs computation of the regularized solution norm and the corresponding residual norm and selects the parameter which maximizes the product of these norms as a function of the regularization parameter; its main virtue is that it constructs regularized solutions of either large or small norm depending on whether a certain inclusion condition is satisfied or not. MPC however applied to 3D reconstruction problems may fail due to the existence of several local maxima. To overcome this difficulty the authors developed a variant of MPC, the Improved Product Criterion (IMPC), which via a fast and efficient algorithm chooses as regularization parameter the critical point associated with the largest local maximum of the product. In addition as with MPC, IMPC does not depend on user specified input parameters (like subspace dimension or truncating parameter) and requires no *a priori* knowledge of the noise level.

We organize our paper as follows. Section 2 will be devoted to the formulation of the problem and a brief description of the linear sampling method in the electromagnetic context. Subsequently, Section 3 will deal with the improved version of MPC, the IMPC as a parameter choice rule. In particular we will be concerned with theoretical properties on which IMPC relies on as well as with its implementation within the framework of the linear sampling method. In order to show the effectiveness of our method, in Section 4, we will present numerical examples for the case of penetrable three dimensional scatterers and we will compare the reconstructions obtained via IMPC with the ones obtained by means of Morozov's generalized discrepancy principle (GDP). We will finally list our conclusions in Section 5.

2. The linear sampling method

We begin by considering the direct scattering problem of a time harmonic electromagnetic wave by a penetrable isotropic medium $D \subset \mathbb{R}^3$ which can be formulated as the problem of finding an electric field E and a magnetic field H such that $E, H \in C^1(\mathbb{R}^3)$ and

$$\text{curl}E - ikH = 0 \quad \text{and} \quad \text{curl}H + iknE = 0 \quad \text{in } \mathbb{R}^3 \tag{1}$$

where $n \in C^{1,\alpha}(\mathbb{R}^3)$ is a complex valued function with $0 \leq \alpha \leq 1$ and $n(x) = 1$ outside D. The total field is given as

$$E = E^i + E^s \quad \text{and} \quad H = H^i + H^s \tag{2}$$

where E^s, H^s are the scattered fields satisfying the Silver–Müller radiation condition

$$\lim_{r\to\infty}(H^s \times x - rE^s) = 0 \tag{3}$$

uniformly in $\hat{x} = \frac{x}{|x|}$, where $r = |x|$ and the incident field is the plane wave

$$E^i(x) = \frac{i}{k}\text{curlcurl}\, pe^{ikx\cdot d} = ik(d \times p) \times de^{ikx\cdot d}, \tag{4}$$

$$H^i(x) = \text{curl}\, pe^{ikx\cdot d} = ikd \times pe^{ikx\cdot d}, \tag{5}$$

where the wavenumber k is positive, d is a unit vector giving the direction of propagation and p is the polarization vector. The existence and uniqueness of a solution to (1)–(3) can be found in [11]. From the second Stratton–Chu formula it follows that

$$E^s(x) = \frac{e^{ikr}}{r}\left\{E_\infty(\hat{x}, d, p) + O\left(\frac{1}{r}\right)\right\} \quad \text{as } r \to \infty \tag{6}$$

where E_∞ is the electric far field pattern. The inverse medium problem for electromagnetic waves is to determine D from $E_\infty(\hat{x}, d, p)$ for $\hat{x}$, d in the unit sphere Ω, $p \in \mathbb{R}^3$, and different values of k. As indicated in [10], E_∞ is infinitely differentiable as a function of its arguments and as a function of $\hat{x}$ is tangential to the unit sphere Ω.

We now introduce the space

$$L^2_t(\Omega) = \{g : \Omega \to \mathbb{C}^3 |\, g \in L^2(\Omega),\ g \cdot \hat{x} = 0,\ \text{for } \hat{x} \in \Omega\}$$

of tangential L^2 fields in Ω. The electric far-field operator $\mathcal{F} : L^2_t(\Omega) \to L^2_t(\Omega)$ is then defined by

$$(\mathcal{F}g)(\hat{x}) = \int_\Omega E_\infty(\hat{x}, d, g(d))\, ds(d), \quad \hat{x} \in \Omega. \tag{7}$$

Now let $E_\infty(\hat{x}, z, q)$ be the electric far-field pattern of an electric dipole located in $z \in D$ and oriented along q:

$$E_e(x, z, q) = \frac{i}{k}\text{curl}_x\text{curl}_x q\Phi(x, z) \tag{8}$$

$$H_e(x, z, q) = \text{curl}_x q\Phi(x, z) \tag{9}$$

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