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A meshless local Petrov–Galerkin method for the time-dependent Maxwell equations

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ABSTRACT

In this paper, the meshless local Petrov–Galerkin (MLPG) method is employed to solve the 2-D time-dependent Maxwell equations. The MLPG method is a truly meshless method in which the trial and test functions are chosen from totally different functional spaces. In the current work, the moving least square reproducing kernel (MLSRK) scheme is chosen to be the trial function. The method is applied for the unsteady Maxwell equations in different media. In the local weak form, by employing the difference operator for evolution in time and simultaneously in time and space, the semi-discrete and fully discrete schemes are obtained respectively. The error estimation is discussed for both the semi-discrete and fully-discrete numerical schemes for modelling the time-dependent Maxwell equations. We show that provided that the time step size τ is sufficiently small, the proposed scheme yields an error of $\mathcal{O}(\rho^{2(m+1)} + \tau^2)$ in the $\mathcal{L}^2$ norm for the square of error. The new scheme is implemented and the numerical results are provided to justify our theoretical analysis.

1. Introduction

The numerical treatment of partial differential equations (PDEs) has been a very active research area in the last decades. This purpose has been obtained by several approaches for boundary value problems. Some of these strategies are finite difference method (FDM) [1], finite element method (FEM), finite volume method (FVM) [2], spectral method, spectral element method (SEM) [3] and boundary element method (BEM) [4]. Even though, there exist many numerical and analytical techniques for solving engineering problems, the meshless methods [5,6] are receiving attention in the engineering and scientific modelling. The list of prominent meshless methods includes the smoothed particle hydrodynamics (SPH) method [7,8], the diffuse element method (DEM) [9], the element free Galerkin (EFG) method [10], the reproducing kernel particle method (RKPM) [11–14], the HP-cloud method [15], the partition of unity finite element method (PUFEM) [16], etc.

Since the background cells over the entire domain are needed in the integration of the Galerkin weak-form in these methods, the weak form based methods are referred to as pseudo-meshless methods. The meshless local Petrov–Galerkin (MLPG) method of Atluri [17] and his colleagues is developed as a new class of meshless methods that the local sub domains over each node are used instead of the background cells to integrate the weak-form equations. In view of the fact that no

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global integration is involved, the MLPG method is addressed as a truly meshless method and has been successfully applied to find the numerical solution of a wide range of problems in engineering [17–24].

In the current work, the MLPG method is employed to numerical analysis of the two-dimensional unsteady Maxwell equations when the MLSRK approximation is used as trial function.

The Maxwell equations are a set of PDEs published by Maxwell in 1865 [25], which describe how the electric and magnetic fields are related to their sources, charge density and current density. Later, O. Heaviside [26] and W. Gibbs transformed these equations into the today's vector notations.

To describe the electromagnetic wave propagation, under certain assumptions, in three dispersive medias; i.e. cold plasma, Debye medium and Lorentz medium; the unified model for governing equations is considered as:

$$\mu\frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = 0, \quad \text{in } \Omega \times (0, T], \tag{1.1}$$

$$\epsilon\frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} + \mathbf{E} + \mathbf{J}(\mathbf{x}, t) = 0, \quad \text{in } \Omega \times (0, T], \tag{1.2}$$

where $\mathbf{E}$ is electric field, $\mathbf{H}$ is magnetic field, ϵ is permittivity of free space, μ is the permeability of free space and the polarization current $\mathbf{J}$ is as follows:

$$\mathbf{J}(\mathbf{E}, t) = \omega_p^2 \int_0^t e^{-\nu(t-s)}\mathbf{E}(\mathbf{x}, s)\,ds. \tag{1.3}$$

The boundary $\partial\Omega$ of Ω is considered to be perfect conduction boundary which means:

$$\mathbf{n} \times \mathbf{E} = 0, \quad \text{on } \partial\Omega \times (0, T], \tag{1.4}$$

where ω_p is the plasma frequency, ν is the electron-neutral collision frequency and initial conditions are

$$\mathbf{E}(\mathbf{x}, 0) = \mathbf{E}_0(\mathbf{x}), \qquad \mathbf{H}(\mathbf{x}, 0) = \mathbf{H}_0(\mathbf{x}), \quad \text{in } \Omega, \tag{1.5}$$

where $\mathbf{E}_0$ and $\mathbf{H}_0$ are given functions with $\mathbf{H}_0$ satisfying

$$\nabla \cdot (\mu\mathbf{H}_0) = 0, \quad \text{in } \Omega, \qquad \mathbf{H}_0 \cdot \mathbf{n} = 0, \quad \text{on } \partial\Omega. \tag{1.6}$$

One can easily obtain, from (1.1) and (1.6), that

$$\nabla \cdot (\mu\mathbf{H}) = 0,$$

which is a solenoidal condition for magnetic flux density. Also, the second condition of (1.6), together with (1.1) and (1.2), leads to [27,28]

$$\mathbf{H} \cdot \mathbf{n} = 0, \quad \text{on } \partial\Omega, \tag{1.7}$$

which is also a perfect conduct boundary condition.

The efficient and accurate numerical solutions of electromagnetic wave phenomena have been required in many areas of science and engineering such as telecommunication, antenna design, detection of targets and medical imaging. Therefore, the various types of numerical techniques have been employed to solve the Maxwell equations. The earliest and simplest one of them is the finite difference time domain (FDTD) and higher order FDTD schemes [29,30], which are useful for structured grids. Recently, different finite element methods have gained a great attention in solving Maxwell equations [31–34] for both time-harmonic and time-dependent ones. In the past decades, substantial interest have been paid to discontinuous Galerkin (DG) finite element method for the numerical solution of the Maxwell equations in various types of medias. Also, there are other well-known methods which are considered in solving Maxwell equations such as: spectral method [35,36], spectral element method [37–39], meshless radial basis functions [40–42] and etc. Also we refer the interested reader to [43–45] for some research works on meshless techniques for the numerical solution of partial differential equations.

The rest of this paper is organized as follows: the local weak formulation of the studied problem is presented in Section 2. A brief introduction of the MLSRK approximation is introduced in Section 3. The Section 4 is devoted to the numerical implementation of the MLPG method and the numerical integration is performed for the resulted integrals from the local weak forms. In Section 5, the error estimate is discussed for both semi-discrete and fully-discrete schemes. The numerical results are given in Section 6 to demonstrate the effectiveness of the proposed method. Finally, Section 7 completes this paper by the concluding remarks.

2. The MLPG formulation

2.1. Time derivative approximation

To obtain a fully discrete scheme, the time interval $(0, T)$ has been divided into the N uniform subintervals by employing nodes $0 = t_0 \le t_1 \le \cdots \le t_N = T$, where $t_n = n\tau$, then to deal with the time derivatives, the following difference