Journal of Computational and Applied Mathematics 268 (2014) 135–144

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

The dynamics of economic games based on product differentiation

S.S. Askar [a,b,*], Ahmad Alshamrani [a]

[a] Department of Statistics and Operations Researches, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia

[b] Department of Mathematics, Faculty of Science, Mansoura University, Mansoura 35516, Egypt

ARTICLE INFO

Article history:
Received 15 December 2013
Received in revised form 5 February 2014

Keywords:
Discrete dynamical system
Triopoly games
Puu triopoly games
Nash equilibria
Stability
Bifurcations

ABSTRACT

The time evolution of dynamic triopoly games is modeled by a discrete dynamical system obtained by the iteration of a three-dimensional map. We present in this paper four games: a rational Cournot triopoly, a rational Bertrand triopoly, a Puu triopoly with quantity competition, and a cooperative Cournot triopoly game. For each game, the Nash equilibrium of the game is computed and complete analytical and numerical studies of the stability conditions for the fixed points, which are the Nash equilibria, are given. The analysis of bifurcations which cause qualitative changes in the behavior of games and cause loss of stability of Nash equilibrium is investigated through numerical explorations.

1. Introduction

Long time ago, oligopoly theory or one can say the economical competition among few firms goes back to Cournot [1] who introduced a model of imperfect competition between firms, and by now it has become a central concept in the field of economical market. Cournot has suggested quantities as strategic variables in which firms may play in market. In 1883, Bertrand [2] has proposed prices as other strategic variables by which some firms may prefer. Since the appearance of these two important papers, a huge number of papers have extended these observations. Quite early studies on such economical competition have suspected to lead to complex dynamic behavior such bifurcation and chaos. In [3], Askar has shown that such complex dynamics have arisen in a simple monopoly model. Depending on a demand function that has no inflection points, such complex behaviors have been reported in [4]. The complex characteristics of a Cournot duopoly game with concave demand function have been studied by Askar [5].

It has been reported elsewhere [6–8] that quantities induce a lower degree of competition if commodities are substitutes. Therefore, if firms are free to choose their strategic variables, they definitely prefer to perform with quantities rather than prices. There are also another type of strategic variables which are mixed of quantities and price variables. The games governed by this type of variables are called Cournot–Bertrand games. To the best of our knowledge, Shubik [9] was the first who systematically investigated duopoly settings with firms choosing quantities and prices at the same time. However Shubik has formulated the analytical framework for the analysis of this kind of games; he was not able to derive the equilibrium solution of these games. Since then, the literature has reported a lower number of papers in which the equilibrium solution (Nash equilibrium) has analytically obtained and its characteristic such as stability has been discussed.

* Corresponding author at: Department of Statistics and Operations Researches, College of Science, King Saud University, P.O. Box 2455, Riyadh 11451, Saudi Arabia.

E-mail addresses: s.e.a.askar@hotmail.co.uk, saskar@ksu.edu.sa, sameh_asker@mans.edu.eg (S.S. Askar).

http://dx.doi.org/10.1016/j.cam.2014.02.032
0377-0427/

The Cournot–Bertrand competition requires a certain degree of differentiation between products offered by firms to avoid one firm from dominating the market by its lower price. In [10–13], the authors have argued that in certain cases, the Cournot–Bertrand game may be optimal. Moreover, Tremblay et al. [14] has illustrated that empirical evidence has proven that this kind of competition is abundant. Recently, Tremblay et al. [15] have studied the static prosperities of the Nash equilibrium of a Cournot–Bertrand duopoly according to product differentiation. Naimzada [16] has studied the dynamic prosperities of a Cournot duopoly game with product differentiation using the linearity of demand and cost objective functions.

Bounded rationality and Puu's incomplete information are two different frameworks that have been recently used to study monopoly and duopoly markets. Bounded rational players (firms) update their production strategies based on discrete time periods and by using a local estimate of the marginal profit. With such a local adjustment mechanism, the players are not requested to have a complete knowledge of the demand and the cost functions [17]. All they need to know is if the market will response to small production changes by an estimate of the marginal profit. This adjustment mechanism, which sometimes called myopic [17], has been extensively used by many authors, mainly with continuous time [18]. However, it is argued elsewhere [17] that a discrete time decision process is more realistic since in real economic systems, production decisions cannot be revised at every time instant. On the other hand, Puu [19] has recently introduced the so-called Puu's incomplete information. It has main advantage that it is realistic since a firm does not need to know the form of the profit function to get an estimate of the quantity produced in the next time step. Instead all it needs to know is its profit and the quantities produced in the past two times. Recently, Ahmed et al. [20] have reported that systems based on Puu's techniques are numerically unstable when approaching to the equilibrium position. Moreover such systems have serious instabilities in the case of duopoly. The authors in [20] have modified those systems based on Puu's techniques with a change in the quantities produced by 10% per time step to avoid singularities in such systems.

In this paper, we introduce and study different types of Cournot and Bertrand games with three competitors. Our model generalizes the duopoly model which was introduced in [16] and furthermore we propose and study other types of games. First, we propose a rational triopoly game in which three firms are in competition. We introduce and study Cournot and Bertrand triopoly games separately. Common questions that arise in the study of such competition models are: what is the impact of adding a new competitor? Is the system more stable or more vulnerable? Are the dynamic properties more complex? First, our analysis will point out the differences between the different types of games studied in this paper. Second Puu's triopoly game is introduced and its dynamic characteristics are investigated. Finally, we propose that two firms will cooperate with each other against the third competitor. In this game, the Nash equilibrium is obtained and its complex characteristics are explained numerically.

The paper is organized as follows. In Section 2 a description of Cournot and Bertrand oligopoly games is provided, and the three-dimensional map whose iteration gives the time evolution based on the bounded rationality is defined. The equilibrium points of the maps, which are Nash equilibria, are computed. Then the stability of those equilibrium points is investigated and their complex dynamic characteristics are detected. In Section 3 a Puu triopoly model is introduced and its Nash point is computed and studied together with its local stability and bifurcation. In Section 4 a cooperation between two firms against the third is suggested and the unique Nash point is computed. The stability of this point is investigated and its properties are discussed. We end the paper with some conclusions to show the significance of our results.

2. Rational triopoly games and main results

In this section, we analyze the competition of three firms in a single market. Firms' decision variables are either prices p_i and/or production quantities q_i, $i = 1, 2, 3$. To introduce our triopoly model, we consider a generalized version of the model proposed by Singh and Vives [6] for the triopoly game. The following representative utility function is a quadratic function of three differentiated products, q_1, q_2 and q_3. It is given by,

$$U(q_1, q_2, q_3) = a(q_1 + q_2 + q_3) - \frac{1}{2}(q_1^2 + dq_2^2 + q_3^2) - (q_1q_2 + dq_1q_3 + q_2q_3). \tag{1}$$

Utility maximization yields the inverse demand functions, i.e., the price of good as a function of quantities produced by the three firms. It is given as follows:

$$\begin{aligned} p_1 &= \frac{\partial U}{\partial q_1} = a - q_1 - q_2 - dq_3 \\ p_2 &= \frac{\partial U}{\partial q_2} = a - q_1 - dq_2 - q_3 \\ p_3 &= \frac{\partial U}{\partial q_3} = a - dq_1 - q_2 - q_3 \end{aligned} \tag{2}$$

where $a > 0$ and $0 \leq d \leq 1$. The parameter d represents the degree of product differentiation or product substitution. If $d = 1$, the three inverse demand functions become identical, which is the case of homogeneous goods. If $d = 0$, this implies that the market has three monopolistic firms. By assuming negative values of the parameter d implies that complementarity between the three firms. Here, we consider the following games: