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Elastoplastic analysis by active macro-zones with linear kinematic hardening and von Mises materials

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ABSTRACT

In this paper a strategy to perform elastoplastic analysis with linear kinematic hardening for von Mises materials under plane strain conditions is shown. The proposed approach works with the Symmetric Galerkin Boundary Element Method applied to multidomain problems using a mixed variables approach, to obtain a more stringent solution. The elastoplastic analysis is carried out as the response to the loads and the plastic strains, the latter evaluated through the self-equilibrium stress matrix. This matrix is used both, in the predictor phase, for trial stress evaluation and, in the corrector phase, for solving a nonlinear global system which provides the elastoplastic solution of the active macro-zones, i.e. those zones collecting bem-elements where the plastic consistency condition has been violated.

The simultaneous use of active macro-zones gives rise to a nonlocal approach which is characterized by a large decrease in the plastic iteration number, although the proposed strategy requires the inversion and updating of Jacobian operators generally of big dimensions. A strategy developed in order to reduce the computational efforts due to the use of this matrix, in a recursive process, is shown.

1. Introduction

The present paper shows a Symmetric Galerkin Boundary Element Method (SGBEM) using a mixed variables multidomain approach [1] in initial strain elastoplastic 2D problems, in the hypothesis of von Mises materials, associated flow rule, plane strain state and linear kinematic hardening behaviour.

In the past, in the BEM context, the elastoplastic analysis was dealt with using the collocation approach: we can mention the papers by Aliabadi and Martin [2] dealing with the contact problem, Hatzigeorgeou and Beskos [3] in dynamics through 3D inner discretization, Ribeiro et al. [4] through generation of cells introduced during the analysis process, Brebbia et al. [5] and the numerous researches inserted in the relevant references to the authors cited.

Differently, the present approach utilizes the symmetric BEM and starts from the discretization of the domain into substructures, called bem-elements (bem-e), where the plastic strains stored in discrete terms have to be computed. The idea of subdividing the domain into substructures was introduced in the SGBEM by Maier et al. [6] through a variational approach. Subsequently Gray and Paulino [7] utilized substructuring in potential problems; Layton et al. [8] proposed a formulation dividing the body into macroelements, each of which is governed by boundary quantities only, obtaining inside a condensation process a system having some non-symmetric blocks; Ganguly et al. [9] presented an entirely symmetric approach for a plane elastic body subdivided by two macroelements, discretized along the boundary and characterized by a symmetric solving system having as unknowns the boundary quantities of both the macroelements.

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Panzeca et al. dealt with the same problem by using a strategy connected to the SGBEM variational formulation introduced by Maier and Polizzotto [10] and Polizzotto [11,12], and obtained two different multidomain approaches defined as mixed variable approach [13] and displacement one [14,15], both characterized by strong variable condensation, numerically compared by Terravecchia [16] inside an elastic analysis. Through these strategies the authors obtained an equation system only depending on the interface nodal variables of the contiguous bem-elements.

The mixed variables multidomain approach was developed as a natural evolution of the displacements approach [15], applied by the same research group in some mechanics problems like thermoelasticity [17], the contact-detachment problem [18], the mechanics of quasi-brittle fracture [19] and ideal elastoplasticity [20]. The approaches guarantee the compatibility and equilibrium at all the points of the domain, but only in weighted form along the boundary elements as a consequence of the discretization.

The strategy contemplates the evaluation, for each bem-e, of expressions which relate mechanical and kinematical weighted quantities in the interface boundaries and stresses evaluated at Gauss points to mechanical and kinematical nodal quantities defined in the same interface boundaries, and to the boundary (forces and imposed displacements) and domain (body forces and plastic strains) actions. These expressions, written for every bem-e, are characterized by elastic operators containing the geometry and constitutive data.

The governing equation is obtained by imposing the regularity conditions between bem-elements regarding the kinematical and mechanical quantities both in terms of nodal variables (strong regularity) and in terms of generalized ones (weak regularity).

This procedure uses a self-equilibrium stresses equation [21,22] governing the elastoplastic problem and relates stresses, valuated at the Gauss points, to stored plastic strains through an influence matrix (self-stress matrix) which is non-symmetric, negative semi-definite and fully-populated.

Inside a step-by-step load process, within which the elastoplastic analysis is considered path-independent, this matrix makes it possible to perform the predictor phase, and therefore it locates all the active bem-elements, and as a consequence defines the active macro-zones.

Then, in a subsequent phase, the elastic solution is corrected by a return mapping algorithm. The proposed strategy permits the simultaneous correction of the elastic solution in all the active bem-elements and utilizes the same self-equilibrium stresses matrix, changed in sign, in a nonlinear system having the end-step stresses, the internal variables (back stresses) and the plastic multiplier increments as unknowns. In the present approach the approximate solution of the previous nonlinear problem is obtained by the well-known Newton–Raphson procedure (N–R), utilized for Bem formulations by several authors [23–27].

The strategy shows considerable computational advantages: the low number of unknowns in the elastic analysis, the reduced number of plastic iterations (three or four for any load step), and the updating of the plastic strains in the bem-elements through the employment of a nonlocal strategy where the plastic strains are stored simultaneously in all the active bem-elements. The operative difficulties arise from the large dimension of the Jacobian operator, but in this paper for its inversion a strategy to reduce the computational efforts is shown.

Moreover the present approach offers the advantages of updating the nodal unknowns caused by plastic actions only at the end of single load step and directly obtaining a path-independent elastoplastic solution.

This paper shows some characteristics of the elastoplastic analysis dealt with the SGBEM using the active macro-zones, in two previous papers by some of the present authors. In the first paper [28] the elastoplastic analysis is carried out through the so-called "displacement method" of the SGBEM, where the constitutive model of the material is elastic-perfectly plastic. In the second one [29] the elastic-perfectly plastic analysis is dealt with as a limit analysis, performed in a canonical form as a convex optimization problem with quadratic constraints, in terms of discrete variables, and implemented with the Karnak.sGbem code coupled with the optimization toolbox by MatLab.

Some examples are shown using the Karnak.sGbem analysis code [30], by which it was possible to make some comparisons with other approaches for the purpose of showing the effectiveness of the proposed method.

2. Self-stress equation via multidomain SGBEM

This section shows a detailed description of the procedure utilized to obtain, using the SGBEM applied to multidomain problems, an elasticity equation connecting the stresses to the plastic strains through a stiffness matrix involving all the bem-elements of the discretized system.

Consider the classical Somigliana Identities (S.Is.):

$$\mathbf{u} = \int_\Gamma \mathbf{G}_{uu}\mathbf{f}\,d\Gamma + \int_\Gamma \mathbf{G}_{ut}(-\mathbf{u})\,d\Gamma + \int_\Omega \mathbf{G}_{u\sigma}\boldsymbol{\varepsilon}^p d\Omega \tag{1a}$$

$$\mathbf{t} = \int_\Gamma \mathbf{G}_{tu}\mathbf{f}\,d\Gamma + \int_\Gamma \mathbf{G}_{tt}(-\mathbf{u})\,d\Gamma + \int_\Omega \mathbf{G}_{t\sigma}\boldsymbol{\varepsilon}^p d\Omega \tag{1b}$$

$$\boldsymbol{\sigma} = \int_\Gamma \mathbf{G}_{\sigma u}\mathbf{f}\,d\Gamma + \int_\Gamma \mathbf{G}_{\sigma t}(-\mathbf{u})\,d\Gamma + \int_\Omega \mathbf{G}_{\sigma\sigma}\boldsymbol{\varepsilon}^p d\Omega. \tag{1c}$$