Journal of Computational and Applied Mathematics 264 (2014) 131–138

Contents lists available at ScienceDirect

Journal of Computational and Applied Mathematics

journal homepage: www.elsevier.com/locate/cam

Elastic analysis of a circumferential crack in an isotropic curved beam using the modified mapping–collocation method

Aydin Amireghbali [a], Demirkan Coker [a,b,*]

[a] Department of Aerospace Eng., Middle East Technical University, Ankara, Turkey
[b] METU Center for Wind Energy (METUWIND), Ankara, Turkey

ARTICLE INFO

Article history:
Received 13 February 2013
Received in revised form 24 December 2013

Keywords:
Modified mapping–collocation (MMC) method
Linear elastic fracture mechanics (LEFM)
Stress intensity factor (SIF)
Circumferential crack
Curved beam
Complex analysis

ABSTRACT

The modified mapping-collocation (MMC) method was applied to the boundary value problem (BVP) of a circumferential crack in an isotropic elastic curved beam subjected to pure bending moment loading. The stress correlation technique is then used to determine opening and sliding mode stress intensity factor (SIF) values based on the computed stress field near the crack tip. The MMC method aims at solving two-dimensional BVP of linear elastic fracture mechanics (LEFM) circumventing the need for direct treatment of the biharmonic equation by combining the power of analytic tools of complex analysis (Muskhelishvili formulation, conformal mapping, and continuation arguments) with simplicity of applying the boundary collocation method as a numerical solution approach. A good qualitative agreement between the computed stress contours and the fringe shapes obtained from the photoelastic experiment on a plexiglass specimen is observed. A quantitative comparison with FEM results is also made using ANSYS. The effect of crack size, crack position and beam thickness variation on SIF values and mode-mixity is investigated.

1. Introduction

In metal components with a curved geometry, cracks initiate in the radial direction. However, in heterogeneous structures such as curved laminated composites, cracks occur circumferentially [1]. Even though significant amount of work has been carried out on radial cracks in the literature, sufficient attention has not been paid to the analysis of circumferential cracks. In contrast to radial cracks, the curved crack geometry, independent of applied loading, leads to mixed-mode fracture consisting of opening mode (mode-I) and sliding mode (mode-II).

The modified mapping–collocation (MMC) method [2] was introduced in 1970 to treat two-dimensional fracture mechanics problems for isotropic problems and later developed further for orthotropic cases [3]. In the 1970s the method was applied to a range of two-dimensional problems, all with radial (and therefore straight) cracks; a radial crack in a circular ring [4], a radial crack in a segment of a circular ring [5] and radial cracks emanating from both the inner and outer surfaces of a circular ring [6]. In all these cases mode-I fracture of radial cracks were investigated.

In the present study, the MMC method is applied in order to compute the stress field in an isotropic curved beam with a circumferential crack subjected to pure bending moment (Fig. 1). Thereafter, the stress correlation [7] technique is incorporated in the MMC method to calculate the SIF values for both the opening and sliding modes from the stresses

* Corresponding author at: Department of Aerospace Eng., Middle East Technical University, Ankara, Turkey. Tel.: +90 3122104253; fax: +90 3122104250.
E-mail addresses: aydin.amireghbali@gmail.com (A. Amireghbali), coker@metu.edu.tr, demir.coker@gmail.com (D. Coker).

0377-0427/$ – see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.cam.2013.12.053

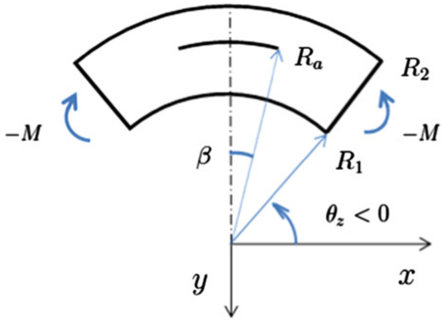

Fig. 1. Curved beam with circumferential crack.

obtained at the vicinity of the crack tip. A parametric study of the effect of crack size, crack position and beam thickness on SIFs and their fraction (mode-mixity) is presented.

2. Theory

The analytic basis of the MMC method is mainly the comprehensive work of N.I. Muskhelishvili [8], in which his complex representations for boundary condition equations besides conformal mapping and continuation (or extension) arguments of both Muskhelishvili and Kartzivadze are introduced and applied to various two-dimensional linear elastic fracture mechanics (LEFM) problems. The numerical part uses the boundary collocation method to solve the overdetermined linear system of equations supplied by the analytic part in a least-square sense.

2.1. Muskhelishvili's complex formulation

According to G.V. Kolosov's formulation [8], only two complex analytic functions (e.g. $\phi(z)$ and $\psi(z)$) are needed in order to describe the stress field in a two-dimensional elastic body:

$$\sigma_y + \sigma_x = 4\Re\{\phi'(z)\} \tag{1}$$

$$\sigma_y - \sigma_x + 2i\sigma_{xy} = 2[\bar{z}\phi''(z) + \psi'(z)]. \tag{2}$$

Further, Muskhelishvili [8] states the boundary condition equations in terms of $\phi(z)$ and $\psi(z)$. His complex formulation for the stress boundary condition equations is:

$$N - iT = \phi'(z) + \overline{\phi'(z)} - [\bar{z}\phi''(z) + \psi'(z)]e^{2i\theta} \tag{3}$$

where N is the normal stress component of boundary traction and T is the tangential component. Also the local force boundary condition equation is given [8] by:

$$(F_x + iF_y)_{on\ AB} = b.i[\phi(z) + z\overline{\phi'(z)} + \overline{\psi(z)}]_{z_A}^{z_B} \tag{4}$$

where F_x and F_y are the local force vector components on segment AB on the boundary and b is the beam depth in the direction perpendicular to the complex z-plane. However A and B do not refer to any particular point on the boundary, once point A is selected on the boundary, point B is chosen such that when *moving* from A to B, the body appears on the left side of the segment AB.

The real power of complex analysis approach to the problem appears when investigating cracked bodies. Finding a proper function which maps the image plane (ζ-plane) into physical plane (z-plane) provides the opportunity of applying the theorems and tools of complex analysis.

2.2. Laurent theorem and series

According to the Laurent theorem, for any function (e.g. $\phi(\zeta)$) that is analytic on an annulus **R**, centered at $\zeta = 0$, there exists a unique power series expansion of the form:

$$\phi(\zeta) = \sum_{n=-\infty}^{+\infty} a_n \zeta^n \tag{5}$$

(namely a Laurent series) which converges to that function on the region **R**.