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Adaptive moving grid methods for two-phase flow in porous media

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ABSTRACT

In this paper, we present an application of the moving mesh method for approximating numerical solutions of the two-phase flow model in porous media. The numerical schemes combine a mixed finite element method and a finite volume method, which can handle the nonlinearities of the governing equations in an efficient way. The adaptive moving grid method is then used to distribute more grid points near the sharp interfaces, which enables us to obtain accurate numerical solutions with fewer computational resources. The numerical experiments indicate that the proposed moving mesh strategy could be an effective way to approximate two-phase flows in porous media.

1. Introduction

Various models of two-phase flows have been widely used in many fields, such as power generation and chemical and petroleum exploitation. Flows of such type are important for the design of steam generators, internal combustion engines, jet engines, refrigeration systems, pipelines for transport of gas and oil mixtures, etc.

Many numerical methods have been applied to solve two-phase flow, including conventional finite difference methods [1,2], discontinuous Galerkin methods [3–5], mixed finite element methods [6–8], higher-order finite element methods [9,10], finite element methods with adaptive mesh refinements [11–13], lattice Boltzmann methods [14,15] and cell-centered finite difference methods with multipoint flux approximation [16–18].

There are two main approaches to model two-phase flow in porous media. The first one solves the balance equations individually for each of the fluids, while the second involves manipulation and combination of those equations into several alternate forms with various choices of primary dependent variables. For the latter method, the choice of equation form and the primary solution defined by variables have considerable implications for the mathematical analysis and the numerical method used to solve these equations. When the second approach above is applied, the governing equations are usually written in terms of a fractional flow formulation, i.e., in terms of the saturation and the global pressure. The main reason for this fractional flow approach is that one can then use efficient numerical methods to take advantage of many physical properties inherent in the flow equations.

Some relevant works choose global pressure as one of the variables of the fractional flow formulation; see, e.g., [19–21] and the references therein. It has been demonstrated that this fractional flow approach is far more efficient than the original

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two-pressure approach from the computational point of view [21]. However, when the mixed finite element (MFE) method is employed, this kind of fractional flow formulation becomes inconsistent [22,20,23–25]. The reason is that the MFE method requires the primary variable and its derivative to be continuous at the grid–block interface, while the global pressure is discontinuous in heterogeneous media [26] due to saturation discontinuity. To avoid the drawback of the fractional flow implementation, Hoteit and Firoozabadi [27] provide a new hybrid MFE formulation in two-dimensional (2D) and three-dimensional (3D) heterogeneous media. Instead of using the global pressure, they employ the wetting-phase pressure as a primary variable, as the wetting-phase pressure is always continuous as long as none of the phases is immobile. Such formulation has also been applied in more complicated three-phase flow problems by Sun and Firoozabadi [10].

The most important characteristic of two-phase flow in porous media is the existence of sharp saturation fronts, and sometimes saturation jumps, in addition to permeability and porosity jumps that separate different media in heterogeneous media cases. The main challenge lies in the fact that extremely fine meshes are required over thin fronts or jumps of the physical domain in order to produce physically correct results. While using uniform meshes in such cases is quite ineffective, adaptive mesh refinement can reduce the computational costs greatly. However, mesh refinement increases the number of total elements, requiring a storage reallocation process, which is quite complex for programming. In this work, instead of local mesh refinement, we extend the moving mesh methods developed in [28,29,12] to increase the numerical efficiency. The method can cluster more grid points to the areas around the sharp interfaces without changing the mesh structure.

The paper is organized as follows. First, we will review the governing equations of two-phase flow, and the new formulation proposed by Hoteit and Firoozabadi [27]. We will then use an iterative implicit pressure explicit concentration (IMPEC) scheme based on the mixed finite element–finite volume (MFE–FV) method [27,10,30] to discretize the governing equations. The moving mesh method will be introduced in Section 4. Finally, several one-dimensional and two-dimensional numerical experiments are carried out to demonstrate the effectiveness of the proposed scheme.

2. Mathematical formulation

Two-phase flow in porous media is usually modeled from a representative elementary volume (REV) scale or Darcy scale point of view. In the Darcy scale, two-phase flow modeling equations can be broken into two parts. The first part is the conservation law, which is exact. For the wetting phase, the mass conservation law reads as

$$\phi \frac{\partial(\rho_w S_w)}{\partial t} + \nabla \cdot (\rho_w v_w) = \hat{F}_w = F_w \rho_w, \tag{2.1}$$

where ϕ is the porosity of the medium, and S_w, F_w, v_w, and ρ_w are the saturation, external volumetric flow rate, volumetric velocity, and density of the wetting phase, respectively. Assuming an incompressible fluid, (2.1) is simplified to the conservation of volume:

$$\phi \frac{\partial S_w}{\partial t} + \nabla \cdot v_w = F_w. \tag{2.2}$$

Similarly, the conservation of volume for the incompressible non-wetting phase yields

$$\phi \frac{\partial S_n}{\partial t} + \nabla \cdot v_n = F_n, \tag{2.3}$$

where S_n, F_n, and v_n are the saturation, external volumetric flow rate, and volumetric velocity of the non-wetting phase, respectively.

To close the system, we need constitutive equations to link the Darcy velocity with pressure, which are given by Darcy's law:

$$v_\alpha = -\frac{k_{r\alpha}}{\mu_\alpha} \boldsymbol{K} (\nabla p_\alpha + \rho_\alpha g \nabla z), \quad \alpha = n, w, \tag{2.4}$$

where the subscripts n and w denote the non-wetting and wetting phases, respectively, $\boldsymbol{K}$ is the absolute permeability tensor, g is the gravity acceleration constant, z is the depth, and p_α, $k_{r\alpha}$, and μ_α are the pressure, relative permeability, and viscosity of phase α, respectively. By definition, the summation of the two saturations must be unity:

$$S_n + S_w = 1. \tag{2.5}$$

In porous media, surface tension and wettability usually cause significant capillary pressure (i.e., the difference of the pressure in non-wetting and wetting phases). Here, we assume that the capillary pressure is a given function of saturation:

$$p_n - p_w = p_c = p_c(S_w). \tag{2.6}$$

Hoteit and Firoozabadi [27] provided a mixed formulation using the wetting-phase pressure as a primary variable. The flow potential Φ_α of the α-phase and the capillary potential Φ_c are defined as follows:

$$\Phi_\alpha = p_\alpha + \rho_\alpha g z, \quad \alpha = n, w, \tag{2.7}$$

$$\Phi_c = \Phi_n - \Phi_w. \tag{2.8}$$

The total velocity v_t can be written in form

$$v_t = v_n + v_w = -\lambda_n \boldsymbol{K} \nabla \Phi_n - \lambda_w \boldsymbol{K} \nabla \Phi_w, \tag{2.9}$$

where $\lambda_\alpha = k_{r\alpha}/\mu_\alpha$ is the mobility of the α-phase.