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A boundary condition based deconvolution framework for image deblurring☆

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HIGHLIGHTS

- Various BC based deconvolution methods are re-derived.
- We propose a repeated BC based deconvolution method.
- The repeated BC can outperform undetermined BC in some cases.

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ABSTRACT

In image deconvolution, various boundary conditions (BC) based deconvolution methods have been proposed to reduce boundary artifacts. However, most of them are not considering the accuracy of BC due to computation limitation. In this paper, we propose a BC based deconvolution framework, which considers the convolution matrix as a product of partial convolution matrix and boundary condition matrix. By computing the adjoint matrix of boundary condition matrix, we can solve this large linear system with conjugate gradient algorithm. With this framework, we can easily derive two efficient non-blind image deconvolution algorithms, which treat the borders of image as repeated instances of the edge pixel values and unknown variables, respectively. Experiments on synthetic data and real data are both presented to show the performance of various BCs. Our conclusion is that undetermined BC usually has the best performance, and repeated BC outperforms undetermined BC if the latent image has high local similarity around the boundary.

1. Introduction

In digital image processing, the general, discrete model for a linear degradation caused by blurring and additive noise can be given by the following superposition summation [1],

$$y(i,j) = \sum_k \sum_l h(i,j,k,l)x(k,l) + n(i,j), \tag{1.1}$$

where x is an original image, h is the point spread function (PSF) of imaging system, and y represents the degraded image which is acquired by the imaging system. In this formulation, n represents an additive noise introduced by the system, and is assumed to be a zero mean Gaussian distributed white noise. Let x and y be written as vectors, arranged in column

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Fig. 1. An example of boundary artifacts. (a) The original image with a field of view shown in the white box. (b) Blurred image using a uniform kernel of size $9 * 9$, where the missing boundary values are set to zeros. (c) Deblurred image with severe boundary artifacts.

lexicographic ordering, then (1.1) can be represented in terms of a matrix–vector formulation,

$$y = Hx + n, \tag{1.2}$$

where H represents the convolution matrix defined by the PSF.

Image deconvolution aims at recovering x from y. The problem is called blind deconvolution if both PSF and image are unknown, or non-blind deconvolution if only the image is unknown. Image deconvolution has been widely applied in many fields, such as astronomical imaging, remote sensing, medical imaging, and digital photography. In this paper, we focus on non-blind deconvolution problem.

Recent research shows that, for atmospherically degraded blur [2] and motion blur [3–5], the PSF can be approximated as a spatially invariant kernel function and hence, the image degradation model has the form of (see Eq. (2.2) in [6])

$$y(i,j) = h(i,j) * x^e(i,j) + n(i,j), \tag{1.3}$$

where $*$ denotes a partial 2-D convolution which computes only the pixels inside Field of View (FOV), and x^e denotes an expanded version of x with the boundary information outside FOV. Since the outside information of FOV is unavailable, some assumptions on the outside values are needed to estimate x. These assumptions are called the boundary conditions [6].

Due to lack of boundary information that is used to produce the blurred image, in most cases, it is impossible to estimate an accurate solution from the observation data. The missing boundary values usually cause serious ringing artifacts around the boundary of restored image, and would propagate it throughout the entire image if image boundary is not well treated. Fig. 1 shows an example of boundary artifacts with zero Dirichlet BC, which assumes the missing boundary values of x are zeros. To reduce boundary artifacts, a simple and effective way is to make the BC as accurate as possible. However, previous BCs, such as zero Dirichlet, periodic, reflective [6] and anti-reflective BC [7], are designed for computation purpose without fully considering the accuracy of BC, and therefore usually introduce boundary artifacts.

As pointed out in [8], the imposed BC has a substantial impact in two directions: (a) precision of the reconstruction especially close to boundaries (presence of ringing effects); (b) cost of computation for recovering the "true" image from a blurred one with or without noise. In this work, we propose a simple BC based deconvolution framework for image deblurring, which usually satisfies requirement (b) and therefore gives us more freedom to consider requirement (a). Unlike the previous algebraic frameworks [6,9,10,8] which represent the convolution matrix H of (1.3) as a summation of several square matrices, this framework simply represents the convolution matrix H of (1.3) as a product of two non-square matrices, a partial convolution matrix and a boundary condition matrix. Given a proper boundary condition, we can easily derive an iterative deconvolution algorithm that not only introduces very little boundary artifacts but also can be implemented efficiently. Incorporated with the state-of-the-art regularization priors, the resulting algorithms can yield a visually pleased image without noticeable noise and boundary artifacts.

1.1. Existing BCs

Generally, the missing boundary information can be estimated by extrapolating the available image data. Several extrapolation methods have been proposed by defining different BCs, including zero Dirichlet, periodic, improved periodic [11], reflective (also known as Neumann [6]), anti-reflective [7,12], and synthetic BC [9].

Reflective BC treats the scene outside the FOV as a mirror reflection of the scene inside the FOV, and it therefore preserves the continuities at the boundary (rigorously, there are two kinds of Reflective BC, one is half-sample symmetric BC [6], and the other is whole-sample symmetric BC [10]). However, reflective BC may make sense only when there are significant features that overlap the edge of the viewable region. Unlike reflective BC, anti-reflective BC preserves not only the continuity of image but also the continuity of normal derivative. In the case of reflective or anti-reflective BC, if the PSF is strongly symmetric, the resulting convolution matrix H can be diagonalized by 2D discrete cosine transform or 2D discrete sine transform, respectively. This good property allows efficient implementation of direct filtering type methods, such as spectral