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An optimal replacement policy for a degenerative system with two-types of failure states

Guo Qing Cheng *, Ling Li

School of Information Engineering, Jingdezhen Ceramic Institute, Jingdezhen, China

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ABSTRACT

In this paper, a deteriorating simple repairable system with two-types of failure states, is studied. Assume that the system after repair cannot be as good as new, and the deterioration is stochastic. Under these assumptions, a new general monotone process model for the degenerative system is introduced which is a generalization of the α-series process model. We use a replacement policy N based on the failure number of the system, then our aim is to determine an optimal replacement policy N^* such that the average cost rate is minimized. We derive the optimal replacement policy analytically or numerically. Finally, we provide a numerical example, and carry through some discussions and sensitivity analysis of the model in this paper.

1. Introduction

In maintenance problems, most research work so far assumes that a failed system after repair will be as good as new, this is the perfect repair model. In practice, it is not always the case, most repairable systems are deteriorating due to accumulative wear. Barlow and Hunter [1] proposed the minimal repair model by assuming that a failed system after repair will function again, but with the same failure rate and the same effective age as at the time of failure. Later on, Brown and Proschan [2] first suggested an imperfect repair model in which the repair was perfect with probability p, or minimal with probability $1 - p$. Much research has been completed by Park [3], Kijima [4] and others along this direction.

For a deteriorating simple repairable system, its successive working times of the system after repair may become shorter and shorter while the consecutive repair times of the system may become longer and longer. Ultimately, such a system cannot work any longer. Neither can it be repaired any more. To model such a deteriorating system, Lam [5,6] first introduced a geometric process repair model in which he studied two kinds of replacement policy, one based on the working age T of the system and the other based on the number of failures N of the system. The objectives were to choose optimal replacement policies T^*, and N^*, respectively, such that the long-run expected cost per unit time is minimized. Under some conditions, Lam [6] proved that the optimal policy N^* is better than the optimal policy T^*. Finkelstein [7] presented a general repair model based on a scale transformation to generalize Lam's work. Zhang [8] generalized Lam's work with a bivariate replacement policy (T, N) under which the system is replaced at the working age T or at the time of the Nth failure, whichever occurs first. The objective was to choose the optimal replacement policy $(T, N)^*$ such that the long-run average cost per unit time is minimized. Other works on the geometric process model in maintenance analysis include Stadje and Zuckerman [9], Lam [10], Stanley [11], and Zhang et al. [12].

Recently, Braun et al. [13] proposed an alternative model, the α-series process which is tractable and easy to use just like the geometric process. Furthermore, Braun et al. [14] shows that the increasing geometric process grows at most logarithmically in time while the decreasing geometric process is almost certain to have a time of explosion. The α-series process

* Corresponding author. Tel.: +86 13979874723.
E-mail address: cgq686@163.com (G.Q. Cheng).

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grows either as a polynomial in time or exponentially in time. It also shows that, unlike most renewal processes, the geometric process does not satisfy a central limit theorem, while the α-series process does. The α-series process was proposed as a complementary model which can be used in situations where the geometric process is inappropriate. Tang and Liu [15] presented a maintenance model for a deteriorating system by using the α-series process. They gave the long-run average cost per unit time under the replacement policy N, and obtained the analytic expression for the optimal replacement policy.

In most existing models for maintenance problems, including the geometric process and α-series process models, we always assume that a system has only two states: up and down say. However, in many practical cases, a system may have more than two states. For example, an electronic component such as a diode or a transistor, may fail due to a short circuit or an open circuit. In engineering, a failure of machine may be classified by its seriousness, a slight failure or a serious failure. In man–machine system, a failure may be classified by its cause, a manmade mistake or a machine trouble. Such systems or components have one working state and two or more different failure states. We call them multistate systems.

For a deteriorating simple repairable system with $k+1$ states, including k failure states and one working state, Zhang et al. [16] considered a replacement policy N based on the number of failures of the system. They determined the optimal policy N^* such that the long-run expected profit per unit time is maximized and showed that the model for the multistate system forms a general monotone process model which includes the geometric process model as a special case. Lam et al. [17] presented a monotone process model for a one-component degenerative system with $k+1$ states (k failure states and one working state). They showed that the model is equivalent to a geometric process model for a two-state one component system such that both systems have the same long-run average cost per unit time and the same optimal policy. Other works on the multistate repairable system in maintenance analysis include Zhang [18] and Zhang et al. [19].

The objective of this paper is to study a maintenance model for a degenerative system with two-types of failure states and one working state. In Section 2, a new general monotone process model for the degenerative system is introduced. We can find that the α-series process model is a special case of the monotone process model proposed in this paper. In Section 3, a replacement policy N based on the failure number of the system is adopted under which the system will be replaced at the time of Nth failure. An explicit expression of the average cost rate is derived. Then, an optimal replacement policy can be determined analytically or numerically. Finally, an appropriate numerical example is given, and the influence of the parameters of the model on the optimal solution is also considered in Section 4.

2. The system model

For ease of reference, we first state the definitions of α-series process and list the notation to be used in this paper.

Definition 1. A stochastic process $\{X_n,\ n = 1, 2, \ldots\}$ is called an α-series process, if there exists a real α, such that $\{n^\alpha X_n,\ n = 1, 2, \ldots\}$ forms a renewal process. The real α is called the exponent of the process (see Braun et al. [13,14] and Tang and Liu [15] for more details).

Obviously, for an α-series process, if the distribution function of X_1 is F, then the distribution function of X_n will be F_n with $F_n(t) = F(n^\alpha t), n = 1, 2, \ldots$. If $\alpha > 0$, the α-series process $\{X_n,\ n = 1, 2, \ldots\}$ is stochastically decreasing; and when $\alpha < 0$, the α-series process is stochastically increasing; and when $\alpha = 0$, the α-series process reduces to the renewal process.

If $EX_1 = \lambda$, then we have the $EX_n = \frac{\lambda}{n^\alpha}$.

Now, we introduce a monotone process model for a degenerative system with two-types of failure states by making the following assumptions.

Assumption 1. At the beginning, a new system is installed. Whenever the system fails, it will be repaired or replaced by an identical new one.

Assumption 2. Assume that there are three states in the system, i.e., 0, 1, and 2 are respectively denoted as the working state, 1st type failure state, and 2nd type failure state of the system. And assume that the occurrence of a failure state of two types is mutually exclusive and stochastic. If the system fails, then with probability p_i the system will be in state i, $i = 1, 2$, and $p_1 + p_2 = 1$.

Assumption 3. Repair commences as soon as the system fails. The system after repair is not as good as new. The successive working times of the system form a stochastically monotonically decreasing process while the consecutive repair times of the system form a stochastically monotonically increasing process.

Assumption 4. Let X_1 be the operating time of the system after installation. In general, let X_n be the operating time of the system after the $n-1$th repair. Furthermore, let Y_n be the repair time after the nth failure. Now, denote the time of the nth failure by t_n. Let $S(t_n)$ denote the type of the nth failure where $S(t_n) \in \{1, 2\},\ n = 1, 2, \ldots$. X_n, Y_n are conditionally independent given $S(t_n) \in \{1, 2\},\ n = 1, 2, \ldots$. Assume that

$$P(X_1 \le t) = U(t) \tag{1}$$

$$P(X_n \le t \mid S(t_1) = l_1, \ldots, S(t_{n-1}) = l_{n-1}) = U\left(\left(\frac{2}{1}\right)^{\alpha_{l_1}} \cdot \left(\frac{3}{2}\right)^{\alpha_{l_2}} \cdots \left(\frac{n}{n-1}\right)^{\alpha_{l_{n-1}}} t\right) \tag{2}$$

where $l_i \in \{1, 2\}, i = 1, 2, \ldots, n-1$; $\alpha_1 \ge 0, \alpha_2 \ge 0$.