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Optimal control of switched systems and its parallel optimization algorithm

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ABSTRACT

In this paper, a class of optimal switching control problems is considered in which the mode sequence of active subsystems and the number of mode switchings are not pre-specified, and both the switching sequence and the control inputs are to be chosen such that the cost functional is minimized. For solving this problem, we discuss the necessary conditions for optimality and the construction method for suboptimal solutions, and develop a few sufficient conditions of judgment on the optimal or suboptimal solutions of the switched system. According to the sufficient conditions, a parallel computational algorithm is constructed to find optimal or suboptimal solutions. For illustration, two examples are solved using the proposed algorithm.

1. Introduction

Hybrid systems have been an active research topic in the last few years and many results can be found in the control and computer science literature [1]. The earliest result is proposed by Witsenhausen (1966) [2], which proves a maximum principle for hybrid systems with autonomous switching only. Since then, there has been a mounting interest in the theoretical results for optimal control of hybrid systems which are obtained by extending the classical necessary conditions for optimality or the dynamic programming approach [3–7].

As a special type of hybrid system, switched systems have attracted increasing attention (see, for example, [3,4,8–18]) because of their practical significance and theoretical challenge. Many real-world processes, such as biochemical processes, automotive systems, manufacturing processes, aircraft and air traffic control, and switching power converters, can be modeled as switched systems [19,18]. A switched system consists of a number of subsystems and a proper switching law that orchestrates the active subsystem at each switching instant. Optimal control problems of switched systems are one of the most challenging and important classes of problems, since the solutions of both the optimal switching sequence and the optimal continuous inputs have to be determined such that some performance criterion is minimized subject to some constraints on the state and the control variables. The system dynamics may vary significantly before and after every switch. Many results, which report progress on theoretical and computational issues for continuous-time or discrete-time versions of such problems, have appeared in the literature (see, e.g., [8,20,9,21–23,19,24,10–17]). Several successful families of algorithms have already been developed to seek the optimal control [22,24,11–13,25,18].

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It is worth mentioning that Giua et al. [8,20] considered the optimal control of continuous-time switched systems composed of linear and stable autonomous dynamics with a piecewise-quadratic cost function. Furthermore, Egerstedt et al. [11] and Xu and Antsaklis [21–23,19,24] derived the gradient formula of the cost functional and developed the gradient-descent algorithms to find the optimal switching instants. In particular, Xu and Antsaklis [22,23,19,24] gave the values of the first and second derivatives of the cost functional with respect to the switching instants by the calculus of variations. Among them, we mention an approach based on the parameterization of the switching instants [24] and one based on the differentiation of the cost functional [22,23]. These approaches transform the optimal switching control problem to an equivalent one with parameters and fixed switching instants. Then the values of the derivatives of the cost functional with respect to the switching instants are obtained based on the solution of a two-point boundary-value differential algebraic equation which is obtained by the maximum principle [24]. For a class of problems whose solutions are typically characterized by both bang–bang and singular control regimes, Khmelnitsky [25] solved the globally optimal solutions by reducing them to the combinatorial search for the shortest path in a specially constructed graph. Moreover, Teo and Jennings et al. [26–28] proposed a control parameterization technique and the time scaling transform method [28] to find the approximate optimal control inputs and switching instants, which have been used extensively in [18,29–35]. However, these methods require some assumptions about either the fixed number of mode switchings or the pre-specified switching sequence. In contrast, Bengea and Decarlo [16] considered the optimal control problem for a two-switched system, in which they applied the maximum principle to an embedded system governed by a logical variable and a continuous control, and some necessary and sufficient conditions are introduced for optimality.

In this work, we consider a class of optimal switching control problems involving a dynamical system composed of s ($s \geq 2$) subsystems without imposing restrictions on the mode sequence or the number of mode switchings. Based on Bengea's method, we embed s switched subsystems into a larger family using the parameterized method, and discuss the necessary conditions for optimality and the construction method for suboptimal solutions again due to increasing of the number of subsystems. According to the necessary conditions for optimality, a few sufficient conditions are developed for judging optimal or suboptimal solutions of the switched system. These results indicate that one can obtain a solution of the switched system through solving independently each switched subsystem, and if the maximized Hamiltonian function is taken place in only one subsystem, then this solution is also optimal for the switched optimal control problem. If there are more than two subsystems such that their maximized Hamiltonian functions are equal on a set of nonzero measure, then the switched system does not have an optimal solution, and the suboptimal solution can be constructed. In consideration of both the difficulty of finding analytical solutions and the complexity of solution procedure, we provide a computational algorithm for determining optimal or suboptimal solutions on the basis of the parallel principle. Two examples are solved applying the proposed algorithm on a Lenovo DeepComp 1800 PC-cluster Server, and the numerical results show the effectiveness of the proposed algorithm.

The rest of this paper is organized as follows. The switched, embedded and relaxed system involving s subsystems are formulated in Section 2. In Section 3, the relationships between trajectories of the systems and construction method for suboptimal solutions are discussed. The necessary conditions for optimality are studied and a few sufficient conditions are proposed in Section 4. Section 5 constructs a parallel optimization algorithm for solving optimal solutions, while Section 6 illustrates the numerical results. Conclusions are presented in Section 7.

2. Problem formulation

2.1. The switched system

In this section, we shall study a switched dynamical system and its optimal control. Suppose that the switched system is described as follows:

$$\dot{x}(t) = f(t, x(t), u(t), v(t)), \qquad x(t_0) = x_0, \tag{1}$$

where at each $t \in [t_0, t_f]$, $u(t) \in \mathcal{U}_{ad} \subset \mathbb{R}^m$ is the usual control input constrained to the convex and compact set $\mathcal{U}_{ad}$, and $v(t) \in \mathcal{V}_{ad} \subset \{0, 1\}^{s-1}$ is the switching control and constrained to the set $\mathcal{V}_{ad}$ denoted as

$$\mathcal{V}_{ad} = \{e^0, e^1, e^2, \ldots, e^{s-1}\}. \tag{2}$$

Here, e^i is a $(s-1)$-dimensional unit vector such that the i-th component is one, and others are zeros, $i = 1, 2, \ldots, s-1$. e^0 is a zero vector. u and v are both measurable functions. The function $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^m \times \{0, 1\}^{s-1} \to \mathbb{R}^n$, is a real vector function, and it is piecewise continuously differentiable. Note that the state of system described by Eq. (1) does not undergo jump discontinuities. Suppose that the initial constraint set is $\mathscr{T}_0 \times \mathscr{B}_0$ which contains all of the points (t_0, x_0) such that $x(t_0) = x_0$, and the terminal constraint set is $\mathscr{T}_f \times \mathscr{B}_f$ which requires at some time t_f, where $t_0 < t_f$, the point $(t_f, x(t_f)) \in \mathscr{T}_f \times \mathscr{B}_f$, while for $t_0 \leq t < t_f$ the point $(t, x(t)) \notin \mathscr{T}_f \times \mathscr{B}_f$. Then the endpoint constraint set is denoted as

$$\mathscr{B} = \{(t_0, x_0, t_f, x_f) | (t_0, x_0) \in \mathscr{T}_0 \times \mathscr{B}_0, (t_f, x_f) \in \mathscr{T}_f \times \mathscr{B}_f\}, \tag{3}$$

and it is contained in a compact set in $\mathbb{R}^{2n+2}$.