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Global optimization using a genetic algorithm with hierarchically structured population

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HIGHLIGHTS

- A genetic algorithm (GA) with hierarchically structured population is evaluated.
- The GA is applied to solve benchmark unconstrained optimization problems.
- Computational tests evaluate different structures, population sizes and crossover operators.
- The results found are also compared with those found by other methods from the literature.
- The GA outperforms other approaches for the number of function evaluations and success rate.

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ABSTRACT

This paper applies a genetic algorithm with hierarchically structured population to solve unconstrained optimization problems. The population has individuals distributed in several overlapping clusters, each one with a leader and a variable number of support individuals. The hierarchy establishes that leaders must be fitter than its supporters with the topological organization of the clusters following a tree. Computational tests evaluate different population structures, population sizes and crossover operators for better algorithm performance. A set of known benchmark test problems is solved and the results found are compared with those obtained from other methods described in the literature, namely, two genetic algorithms, a simulated annealing, a differential evolution and a particle swarm optimization. The results indicate that the method employed is capable of achieving better performance than the previous approaches in regard as the two criteria usually employed for comparisons: the number of function evaluations and rate of success. The method also has a superior performance if the number of problems solved is taken into account.

1. Introduction

The present paper proposes a genetic algorithm with hierarchically structured population as a method for finding a global minimum of unconstrained optimization problems. To the best of our knowledge, it has not been proposed in the literature approaches based on GA with hierarchical structure to solve unconstrained optimization problems. This paper fills this gap evaluating the hierarchically structured GA with different crossover operators, several population structures and population sizes.

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Global optimization is the computation and characterization of global optimum which can be a minimum or maximum value of non-convex functions in a specific domain [1]. The problem at hand can be unconstrained or constrained, where optimization techniques must find global values without being trapped into local minima or local maxima points [2]. These problems can be mathematically defined as follows:

$$\text{Minimize } f(x)$$
$$\text{subject to } x \in S,$$

where $f(x), x \in S \subseteq R^n$, is a continuous real-valued function and x is a n-dimensional continuous variable vector.

Unconstrained optimization problems are often found in many applications of engineering and applied sciences, and features of function f such as non-convexity, non-linearity and non-differentiability can make it very hard to find global optimal solutions. Therefore, metaheuristic-based methods have been proposed once that they do not require much information regarding the function to be optimized.

Georgieva and Jordanov [3] solved unconstrained optimization problems for benchmark multi-modal functions with dimensions from 100 to150. A hybrid method composed by Genetic Algorithm (GA), low-discrepancy sequences and heuristic rules is used to find attractive regions where solutions are improved by a Nelder–Mead simplex local search. The authors reported results which are competitive with GA, evolutionary programming and differential evolution (DE) methods. Zhao et al. [4] changed the Nelder–Mead method [5] by moving simplex vertices one by one in the same direction until better results were found. There is a restart for non-convex optimization problem where an initial simplex is defined for solutions returned by the first phase of their method. A total of four sets with test problems of dimensions 2–200 were solved. The authors reported that this modification outperforms an original version and other recent approaches from literature.

An Ant Colony Optimization (ACO) algorithm is used by [6] to find global minima. ACO found optimal values for several test problems and its performance was compared with other algorithms from literature. Kao and Zahara [7] proposed a hybrid method where GA is combined with Particle Swarm Optimization (PSO). The GA incorporates crossover operators based on linear combination of two vectors from PSO. First, GA creates an intermediate population where the worst individuals are improved, using PSO in sequence. This PSO–GA has superior performance to find optimal value for unconstrained optimization problem when compared to other methods from literature, namely, hybrid GA, continuous GA, Simulated Annealing and Tabu Search. Comparisons are made over a set of 17 benchmark problems. Shelokar et al. [8] combine PSO with ACO for non-convex optimization problems. PSO is used in a first phase and an ACO local search procedure is applied in a second one. The pheromone mechanism from ACO updates particle positions determined by PSO during the first phase. A total of 14 benchmark problems of different dimensions are solved and analyzed. The method was proved competitive for a set of small dimension test problems and it arises as a viable alternative for sets of moderate to high dimension test problems.

The papers cited so far have criteria to evaluate their approaches that differ from each other such as number of benchmark problems solved and number of independent runs executed. For this reason, it was decided to concentrate the comparison of the GA proposed in this paper with the works described in [9–11]. These papers present a consistent and homogeneous set of criteria used in the comparisons, namely, success rate, number of function evaluations and number of executions. Additionally, great part of the computational tests reported are carried out over a common set of recognized benchmark problems proposed in [12]. A last argument supporting this choice is that the comparisons encompass methods which belong to a same family, i.e. metaheuristics.

Kaelo et al. [9] evaluate modifications in crossover rules and the use of local search for real coded GA. A combination of crossovers and a probabilistic adaptation of crossover rules are described. A local search improvement procedure is employed by GA every time a best individual is found. A total of five new versions of real coded GA are established and compared with each other. Ali et al. [10] introduce several remedies for slow convergence in a PSO algorithm. These remedies include randomly choosing the best position for worst particles from the best position for best particles, and to update the current particles taking information from the improvement done over the best particle found. The PSO approaches proposed are evaluated and also compared with a differential evolution algorithm. Ali et al. [11] developed a simulated annealing (SA) combined with a multi-level single linkage that is a multi-start method. A derivative-free trial point generation scheme is presented that modifies the derivative free pattern search of the multi-level single linkage. The SA presented a competitive performance in terms of the number of function evaluations and success rate.

The GA proposed in this article is conceived as having a population hierarchically structured, where clusters of individuals are topologically distributed following a tree structure. A better performance of GAs with hierarchically structured populations over unstructured ones has been attested in previous experiments concerning applications to different problems (e.g. machine scheduling, asymmetric traveling salesman, capacitor placement). França et al. [13] first introduce a hybrid population approach for a memetic algorithm (MA) where individuals are hierarchically structured in a ternary tree. The method also uses neighborhood reduction schemes. Instances for the total tardiness single machine scheduling problem are randomly created, and computational simulations prove that hierarchically structured MA outperformed the unstructured version. A similar approach is applied by França et al. [14] to scheduling a flowshop manufacturing cell problem with sequence dependent family setups. The results revealed that structured GAs and MAs were also superior.

Buriol et al. [15] used a MA with population hierarchically structured to solve instances of the asymmetric traveling salesman problem. The authors compared several population structures, population size and some crossover operators. The MA with population structured in ternary tree had a better performance. A total of 27 instances were solved and MA