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On contingent-claim valuation in continuous-time for volatility models of Ornstein–Uhlenbeck type

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ABSTRACT

The paper addresses the valuation of contingent claims in stochastic volatility models of Ornstein–Uhlenbeck type, stressing the situation when volatility is driven by purely-discontinuous Lévy processes. A reduction series methodology is developed for this purpose which also provides a way for the numerical study of the value-functionals. The methodology is illustrated in the options case and in models based on GIG-distributions; numerical examples are provided. These examples show how the series enable computation accuracies of some three decimal places with just a single digit number of terms.

1. Introduction

Seen from a financial perspective, the paper develops a methodology for the valuation of contingent claims in a class of Lévy models, the models for stochastic volatility of Ornstein–Uhlenbeck type; this development is pursued to the point of being able to provide numerical examples.

Mathematically, the paper originates with our quest for a methodology to handle value functionals arising from multi-dimensional stochastic models. We here concentrate on a class of two-dimensional models, those provided by processes of geometric-Brownian-motion form whose variance is stochastic and modeled by an additional process. Typical instances of these models arise in the financial modeling of stochastic volatility. Volatility is empirically observed to possess characteristic stylized facts such as positivity and mean-reversion, and the use of the above models in this context has a major advantage, namely the comparative ease with which to achieve compliance of the modeling with these stylized facts in an a priori sense, namely by the specific choice of processes modeling variance.

In the present paper variance is chosen to follow processes of Ornstein–Uhlenbeck type; these processes are obtained as integrals with respect to Lévy processes, and they have found broad appeal from the modeling of dams over the modeling of turbulence to financial modeling. As a distinguishing feature, we stress variance dynamics driven by Lévy processes that are orthogonal to diffusions by virtue of being purely-discontinuous; see Section 3 for more details. In this we adopt an approach developed by Barndorff-Nielsen and Shephard (see for example their papers from 2000 and 2001), and we also

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focus on the Lévy processes associated with generic generalized inverse Gaussian processes to make matters concrete and explicit (see Appendix A for the concepts involved).

The valuation of contingent claims in these models thus becomes a question now involving at least two processes, and as a first main contribution of the paper we develop a methodology for this in three stages. The starting point of our methodology is furnished by the identification of the structure of the pertinent class of value functionals (see Section 4), and its guiding theme is how the use of reduction series permits the development of methods in terms of the primitives of the modeling. With the appropriate notion of reduction series reviewed in Section 2, this is addressed in Sections 5 and 6 respectively. The result is a methodological platform for handling the value functionals in these OU-type models, a platform that proceeds solely in terms of the primitives of the modeling, namely the payoff functions of the respective contingent claims and the characteristic function of the Lévy-process driver of the variance dynamics.

While our platform involves moments, and conditions for non-explosion in terms of moments in particular, the thrust of our approach is nevertheless very different from the one developed in [1]; rather it furnishes a further development of one of the approaches to Asian option valuation (see for example [2]). In our approach we also focus on the representation of value functionals by convergent series; from this perspective alone it is therefore complementary to the body of results under the heading of asymptotic methods, citing [3,4] as more recent additions here, and with more than 45 additional references cited in [5].

From the picture that emerges from the examples we considered, the convergence of our valuation series tends to be such, however, that they are to be regarded as almost leading-term asymptotic expansions in their own right.

Indeed, in our examples we focus on the setting required for the calibration of the models and consider option valuation in out-of-the-money situations; in this we work with models associated with generalized Gaussian distributions with model-parameters taken from the literature. We find single digit numbers of terms of the reduction series to enable computation accuracies of at least 3 places after the decimal point; see Sections 7.3 and 7.4.

Two remarks of differing qualities may be in order here to complete this introduction. Firstly, modifications of the modeling will entail modifications of the approach developed in the paper. We demonstrate this with extensions indicated in [6,7]; see Section 8. Secondly, proofs are often relegated to later sections or appendices.

2. Preliminaries on methods: Laguerre reduction series

In this section we collect pertinent results from Schröder [8], [9, Section 3] concerning the Laguerre reduction series used in the current paper. Recall the idea for reduction series associated with orthogonal series in its simplest form is as follows: provide series representations for functionals of stochastic processes or random variables in terms of the moments of these processes or random variables, thus effecting a reduction of the former concepts to the latter.

2.1. Laguerre series

We recall as a first step relevant properties of Laguerre polynomials from Lebedev [10, Section 4] or Thangavelu [11]. The structural setting for these is a generalization of the classical Hilbert spaces of square-integrable functions to a class of spaces of weightedly-square-integrable functions obtained by way of introduction of gamma-densities as kernels, as follows. For any real $\alpha > -1$ let $L^2_\alpha(0,\infty)$ be the Hilbert space of all real-valued functions F on the positive reals $(0,\infty)$ that are *α-square integrable*:

$$\|F\|_\alpha^2 = \int_0^\infty w_\alpha(x)|F|^2(x)\,dx < \infty,$$

with w_α on $(0,\infty)$ given by $w_\alpha(x) = x^\alpha \exp(-x)$, the gamma density with parameter α. Then, $L^2_\alpha(0,\infty)$ carries the bilinear form given by

$$\langle F, G\rangle_\alpha = \int_0^\infty w_\alpha(x)F(x)G(x)\,dx,$$

and an orthogonal base for this Hilbert space $L^2_\alpha(0,\infty)$ is provided, after a Gram–Schmidt orthogonalization of the monomials x^m with integer exponents $m \geq 0$, by the *α-Laguerre polynomials* $L^\alpha_m(z)$. For any integer $m \geq 0$, these are given by

$$L^\alpha_m(z) = \sum_{k=0}^m \alpha_{m,k}\, z^k \quad \text{where } \alpha_{m,k} = ((-1)^k/k!)\binom{m+\alpha}{m-k},$$

and satisfy $\|L^\alpha_m\|^2_\alpha = \Gamma(m+\alpha+1)/m!$. Expressing any F in $L^2_\alpha(0,\infty)$ relative to this base, one obtains its *α-Laguerre series*

$$F = \sum_{m=0}^\infty c_m L^\alpha_m \quad \text{where } c_m = \langle F, L^\alpha_m\rangle_\alpha / \langle L^\alpha_m, L^\alpha_m\rangle_\alpha,$$

for any integer $m \geq 0$; the c_m are the *α-Laguerre coefficients* of this series.