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Numerical solution of a class of mixed two-dimensional nonlinear Volterra–Fredholm integral equations using multiquadric radial basis functions

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ABSTRACT

An efficient numerical method is proposed for solving the mixed nonlinear Volterra–Fredholm two-dimensional integral equations (IEs), using two-dimensional radial basis functions (RBFs). This method is based on interpolation by radial basis functions including multiquadrics (MQs), using Legendre–Gauss–Lobatto nodes and weights. Also a theorem is proved for convergence analysis. Some numerical examples are presented and results are compared with the analytical solution to demonstrate the validity and applicability of the proposed method.

1. Introduction

In general form, a mixed Volterra–Fredholm integral equation can be written as

$$f(s,t) = g(s,t) + \int_0^s \int_\Omega U(s,t,x,y,f(x,y))dydx, \tag{1}$$

where $f(s,t)$ is an unknown scalar valued function defined on

$$D = [0,T] \times \Omega$$

and Ω is a closed subset of $\mathbb{R}^n$, $n = 1, 2, 3$. The function $g(s,t)$ and $U(s,t,x,y,f)$ are given functions defined on D and

$$S = \{(s,t,x,y,f) : 0 \le x \le s \le T, t, y \in \Omega\},$$

respectively [1]. It is obvious that a finite interval $[0,T]$ can be transformed to $[-1,1]$ and without loss of generality, we suppose that $\Omega = [0,1]$. Moreover, we assume $U(s,t,x,y,f) = k(s,t,x,y)[f(s,t)]^p$, p is a positive integer. Various problems in physics, mechanics and biology arise from a nonlinear mixed type Volterra–Fredholm integral equation [2–4]. In fact, few numerical methods have been known for approximating the solution of Eq. (1). Kauthen [5,6] presented a continuous time collocation method for the linear case of Eq. (1) and analyzed the discrete convergence properties. Hacia used a projection method for solving the linear case of Eq. (1), [7,8]. Hadizadeh et al. in [9] obtained a numerical solution of linear Volterra–Fredholm integral equations of mixed type using the bivariate Chebyshev collocation approach. Also Banifatemi et al. [10–13] introduced a method for solving Eq. (1) using two-dimensional Legendre wavelets. RBFs were introduced in [14] and they form a primary tool for multivariate interpolation. Hardy [15] showed that MQs are related to a consistent solution of the biharmonic potential problem and thus it has a physical foundation. Buhmann and Micchelli [16] and Chui et al. [17] have shown that RBFs are related to prewavelets. Also Alipanah and Dehghan [18], used RBFs for the solution

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of a nonlinear integral equation in the one-dimensional case. In this paper, we are concerned with the solution of a class of mixed nonlinear Volterra–Fredholm two-dimensional integral equations using MQs. Also we approximate its associated integrals by the Legendre–Gauss–Lobatto points and weights. Thus, we organized this paper as follows. In Section 2, we describe the two-dimensional MQs interpolation. In Section 3, we introduce the Legendre–Gauss–Lobatto nodes and weights. In Section 4, we implement the problem with the proposed method. In Section 5, we will discuss a convergence analysis for a class of mixed two-dimensional nonlinear Volterra–Fredholm integral equations. Finally, we illustrate some numerical examples to show the efficiency and accuracy of this method. The conclusions are discussed in Section 7.

2. Two-dimensional MQs interpolation

Let $\phi(r)$ be MQ functions and we approximate $f(x, y)$ with interpolation by the function $\phi(r)$ i.e.

$$f(x, y) \simeq \sum_{i=0}^{N} \sum_{j=0}^{M} c_{ij}\phi_{ij}(x, y) = C^T \psi(x, y), \tag{2}$$

where

$$\phi_{ij} = \phi_{ij}(x, y) = \phi(\|(x, y) - (x_i, y_j)\|) = \sqrt{\|(x, y) - (x_i, y_j)\|^2 + c^2}, \tag{3}$$

$$\psi(x, y) = [\phi_{00}, \phi_{10}, \ldots, \phi_{N0}; \phi_{01}, \phi_{11}, \ldots, \phi_{N1}; \ldots; \phi_{0M}, \phi_{1M}, \ldots, \phi_{NM}]^T, \tag{4}$$

and

$$C^T = [c_{00}, c_{10}, \ldots, c_{N0}; c_{01}, c_{11}, \ldots, c_{N1}; \ldots; c_{0M}, c_{1M}, \ldots, c_{NM}]^T. \tag{5}$$

Also (x_i, y_j), $i = 0, \ldots, N$, $j = 0, \ldots, M$ are the Legendre–Gauss–Lobatto nodes [19].

3. Legendre–Gauss–Lobatto nodes and weights

Let $\mathcal{H}_N[-1, 1]$ denote the space of algebraic polynomials of degree $\leq N$, by

$$\langle p_i, p_j \rangle = \frac{2}{2j+1}\delta_{ij}.$$

Here $\langle \cdot, \cdot \rangle$ represents the usual $L^2[-1, 1]$ inner product and $\{p_i\}_{i\geq 0}$ are the well-known Legendre polynomials of order i which are orthogonal with respect to the weight function $w(x) = 1$ on the interval $[-1, 1]$ and satisfy the following formula

$$p_0(x) = 1, \qquad p_1(x) = 1,$$
$$p_{i+1}(x) = \left(\frac{2i+1}{i+1}\right) x p_i(x) - \frac{i}{i+1} p_{i-1}(x), \quad i = 1, 2, 3, \ldots.$$

Next, we let $\{x_j\}_{j=0}^{N}$ as

$$\begin{aligned} &(1 - x_j^2)\dot{p}(x_j) = 0, \\ &-1 = x_0 < x_1 < x_2 < \cdots < x_N = 1, \end{aligned} \tag{6}$$

where $\dot{p}(x)$ is a derivative of $p(x)$. No explicit formula for the nodes $\{x_j\}_{j=1}^{N-1}$ is known. However, they are computed numerically using the existing subroutines.

Now, we assume $f \in \mathcal{H}_{2N-1}[-1, 1]$, we have

$$\int_{-1}^{1} f(x)dx \simeq \sum_{j=0}^{N} w_j f(x_j) \tag{7}$$

where w_j are the Legendre–Gauss–Lobatto weights [20,21].

4. Solution of a class of mixed nonlinear Volterra–Fredholm two-dimensional IEs using MQs

Consider the mixed nonlinear Volterra–Fredholm two-dimensional integral equations given in Eq. (1). In order to use MQs, we first perform the expansion of two variables functions $f(s, t)$ in Eq. (1), using Eq. (4), we get

$$C^T \psi(s, t) = g(s, t) + \int_0^s \int_0^1 U(s, t, x, y, C^T \psi(x, y))dydx. \tag{8}$$