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Checking strong optimality of interval linear programming with inequality constraints and nonnegative constraints

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ABSTRACT

This paper considers optimal solutions of interval linear programming problems, in a unified framework. Necessary and sufficient conditions for checking $(\mathbf{A}, \mathbf{b})$-strong optimality and $(\mathbf{A}, \mathbf{b}, \mathbf{c})$-strong optimality of interval linear programming with inequality constraints are developed. The features of the proposed methods are illustrated by some examples.

1. Introduction

The interval linear programming (IvLP) problems have been investigated by many authors, see, e.g., [1–11], among others. Nevertheless, there are only few results on the issue of optimal solutions for a general IvLP, i.e., where the objective cost vector, the coefficient matrix and the right hand vector are all interval vectors or interval matrices. In this paper, we will introduce some new concepts of optimal solutions of IvLP in a unified framework. Necessary and sufficient conditions for checking some types of optimality are developed.

Let us introduce some notation. The i-th row of a matrix $A \in \mathbb{R}^{m\times n}$ is denoted by $A_{i,\cdot}$, the j-th column by $A_{\cdot,j}$. An interval matrix is defined as

$$\mathbf{A} = [\underline{A}, \overline{A}] = \{A \in \mathbb{R}^{m\times n}; \underline{A} \le A \le \overline{A}\},$$

where $\underline{A}, \overline{A} \in \mathbb{R}^{m\times n}$, and $\underline{A} \le \overline{A}$. Similarly, we define an interval vector as an one column interval matrix

$$\mathbf{b} = [\underline{b}, \overline{b}] = \{b \in \mathbb{R}^{m}; \underline{b} \le b \le \overline{b}\},$$

where $\underline{b}, \overline{b} \in \mathbb{R}^{m}$, and $\underline{b} \le \overline{b}$. The set of all m-by-n interval matrices will be denoted by $\mathbb{IR}^{m\times n}$ and the set of all m-dimensional interval vectors by $\mathbb{IR}^{m}$.

Denote by A_c and A_Δ the center and radius matrices given by

$$A_c = \frac{1}{2}(\underline{A} + \overline{A}), \qquad A_\Delta = \frac{1}{2}(\overline{A} - \underline{A}),$$

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respectively. Then $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$. Similarly, the center and radius vectors are defined as

$$b_c = \frac{1}{2}(\underline{b} + \overline{b}), \qquad b_\Delta = \frac{1}{2}(\overline{b} - \underline{b})$$

respectively. Then $\mathbf{b} = [b_c - b_\Delta, b_c + b_\Delta]$.

Let Y_m be the set of all $\{-1, 1\}$ m-dimensional vectors, i.e.

$$Y_m = \{y \in \mathbb{R}^m | \mid y \mid = e\},$$

where $e = (1, \ldots, 1)^T$ is the m-dimensional vector of all 1's. For a given $y \in Y_m$, let

$$T_y = diag(y_1, \ldots, y_m)$$

denote the corresponding diagonal matrix. For each $x \in \mathbb{R}^n$, we define its sign vector sign x by

$$(\text{sign } x)_i = \begin{cases} 1 & \text{if } x_i \geq 0, \\ -1 & \text{if } x_i < 0, \end{cases}$$

where $i = 1, \ldots, n$. Then we have $|x| = T_z x$, where $z = \text{sign } x \in Y_n$.

For a given interval matrix $\mathbf{A} = [A_c - A_\Delta, A_c + A_\Delta]$, and for each vector $y \in Y_m$ and each vector $z \in Y_n$, we introduce the matrices

$$A_{yz} = A_c - T_y A_\Delta T_z,$$

which means

$$(A_{yz})_{ij} = (A_c)_{ij} - y_i (A_\Delta)_{ij} z_j = \begin{cases} \overline{a}_{ij} & \text{if } y_i z_j = -1, \\ \underline{a}_{ij} & \text{if } y_i z_j = 1. \end{cases}$$

where $i = 1, \ldots, m, j = 1, \ldots, n$. Similarly, for an interval vector $\mathbf{b} = [b_c - b_\Delta, b_c + b_\Delta]$ and for each vector $y \in Y_m$, we define vector

$$b_y = b_c + T_y b_\Delta,$$

which means

$$(b_y)_i = (b_c)_i + y_i (b_\Delta)_i = \begin{cases} \overline{b}_i & \text{if } y_i = 1, \\ \underline{b}_i & \text{if } y_i = -1. \end{cases}$$

where $i = 1, \ldots, m$.

2. Unified optimal solution concepts of IvLP and some preliminaries

Consider an LP problem

$$\min \ c^T x \quad \text{subject to } x \in M(A, b), \tag{1}$$

where $M(A, b)$ is the feasible set characterized by a linear system.

Let $\mathbf{A} \in \mathbb{IR}^{m \times n}$, $\mathbf{b} \in \mathbb{IR}^m$ and $\mathbf{c} \in \mathbb{IR}^n$ be given. By an IvLP problem we mean a family of the LP (1), where $A \in \mathbf{A}$, $b \in \mathbf{b}$, $c \in \mathbf{c}$. We write it in short as

$$\min \ \mathbf{c}^T x \quad \text{subject to } x \in M(\mathbf{A}, \mathbf{b}). \tag{2}$$

By *a realization* we mean a concrete setting (1).

In the IvLP theory, one of the following canonical forms

(A) $M(\mathbf{A}, \mathbf{b}) = \{x \in \mathbb{R}^n; \mathbf{A}x = \mathbf{b}, x \geq 0\}$,
(B) $M(\mathbf{A}, \mathbf{b}) = \{x \in \mathbb{R}^n; \mathbf{A}x \leq \mathbf{b}\}$,
(C) $M(\mathbf{A}, \mathbf{b}) = \{x \in \mathbb{R}^n; \mathbf{A}x \leq \mathbf{b}, x \geq 0\}$

is usually assumed [3,7].

2.1. Optimal solution concepts of IvLP

We first review some concepts briefly. A vector $x \in \mathbb{R}^n$ is called *a weak feasible solution* of the IvLP (2) if it is a feasible solution of the LP (1) for some $A \in \mathbf{A}$, $b \in \mathbf{b}$. A vector $x \in \mathbb{R}^n$ is called *a strong feasible solution* of the IvLP (2) if it is a feasible solution of the LP (1) for each $A \in \mathbf{A}$, $b \in \mathbf{b}$ [5,7]. Recently, new concepts of the optimal solution to IvLP are proposed in a unified framework [12].