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Constructing curves and triangular patches by Beta functions

Licai Chu [a,b], Xiao-Ming Zeng [a,*]

[a] Department of Mathematics, Xiamen University, Xiamen, 361005, PR China

[b] School of Science, Jimei University, Xiamen, 361021, PR China

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ABSTRACT

A class of new basis functions for curve and triangular patch modeling is constructed by means of Beta functions. Based on these basis functions, a new scheme of generating curves and triangular patches is proposed. First we demonstrate that these basis functions have similar properties as those of the Bernstein–Bézier basis functions, such as non-negativity, partition of unity and others. Thus, these basis functions give rise to curve and triangular patch representations with affine invariance, convex hull, symmetry and endpoint interpolation, as well as an evaluation algorithm, which is similar to the de Casteljau's algorithm for Bézier curves and surfaces. In addition, these basis functions have a shape parameter. The shape of the curve or triangular patch can be modified by changing the value of the shape parameter under the same control polygon or control net. The modeling examples illustrate the validity of the new methods.

1. Introduction

The designers performing curve and surface modeling often wish that the basis functions have some desirable properties, such as non-negativity, partition of unity and so on. Because of these properties, one can control the curves/surfaces by manipulating the control points. Probability distributions play a very important role in constructing the basis functions. The Bernstein–Bézier basis functions are taken from the binomial distribution [1,2], and the B-spline basis functions also model a simple stochastic process [3,4]. Due to the desirable properties of the basis functions, The Bézier/B-Spline curves and surfaces are widely used in different fields [5,6]. In addition to these basis functions, various bases based on discrete probability distributions have been proposed. Goldman and Morin constructed two types of bases by using the negative binomial distribution and the Poisson distribution, respectively [7,8]. These basis functions share some important properties with the Bernstein–Bézier basis functions. Due to the infinite number of basis functions, an infinite number of control points should be prepared in the curve and surface modeling using these basis functions, which is not convenient for geometric design.

In this paper, we describe the construction of a class of new basis functions that are related to the Beta functions and the Beta-binomial distribution [9], and apply them to curve and triangular patch modeling. Compared with the basis function of [7,8], there are only a finite number of control points involved in the curve and triangular patch modeling. These new basis functions also share some important properties with the Bernstein–Bézier basis functions. A significant difference between them is that the new basis have a shape parameter. The shape of the curves/surfaces can be modified by changing the value of the shape parameter.

The rest of this paper is organized as follows. Section 2 lists the properties of Beta functions, which will be used in the following sections. Section 3 discusses curve modeling. The definitions and properties of the Beta–Bézier basis functions and

* Corresponding author. Tel.: +86 5922580717.
E-mail addresses: chulc@jmu.edu.cn (L. Chu), xmzeng@xmu.edu.cn (X.-M. Zeng).

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the Beta–Bézier curve are provided. Section 4 discusses the Beta–Bézier triangular patch modeling. Finally, the conclusions are provided in Section 5.

2. The Beta functions and their properties

Let $\alpha, \beta \in \mathbb{R}, \alpha, \beta > 0$, the classical Beta function is defined by

$$\mathrm{B}(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}\mathrm{d}x. \tag{1}$$

The Beta function has the following properties (see [10], p. 457):

(i) Positivity

$$\mathrm{B}(\alpha, \beta) > 0, \quad \forall \alpha, \beta > 0. \tag{2}$$

(ii) Symmetry

$$\mathrm{B}(\alpha, \beta) = \mathrm{B}(\beta, \alpha). \tag{3}$$

(iii) Recursion

$$\mathrm{B}(\alpha+1, \beta) = \frac{\alpha}{\alpha+\beta}\mathrm{B}(\alpha, \beta); \tag{4}$$

$$\mathrm{B}(\alpha, \beta+1) = \frac{\beta}{\alpha+\beta}\mathrm{B}(\alpha, \beta). \tag{5}$$

Let $\alpha, \beta, \gamma \in \mathbb{R}, \alpha, \beta, \gamma > 0$, an extension of (1) to three variables is given by

$$\mathrm{B}(\alpha, \beta, \gamma) = \iint_{(x,y)\in\triangle} x^{\alpha-1}y^{\beta-1}(1-x-y)^{\gamma-1}\mathrm{d}x\,\mathrm{d}y, \tag{6}$$

where $\triangle = \{(x, y) \in \mathbb{R}^2 | x, y \geq 0, x+y \leq 1\}$.

There exists a close connection between $\mathrm{B}(\alpha, \beta, \gamma)$ and the Gamma function

$$\Gamma(\alpha) = \int_0^\infty e^{-x}x^{\alpha-1}\mathrm{d}x \quad (\alpha > 0) \tag{7}$$

as the elegant identity

$$\mathrm{B}(\alpha, \beta, \gamma) = \frac{\Gamma(\alpha)\Gamma(\beta)\Gamma(\gamma)}{\Gamma(\alpha+\beta+\gamma)} \tag{8}$$

reveals (see [11]).

From (6) and (8) or see [12,13], it is obvious that the Beta function with three variables also has the properties as follows:

(i) Positivity

$$\mathrm{B}(\alpha, \beta, \gamma) > 0, \quad \forall \alpha, \beta, \gamma > 0. \tag{9}$$

(ii) Symmetry

$$\begin{aligned}\mathrm{B}(\alpha, \beta, \gamma) &= \mathrm{B}(\alpha, \gamma, \beta) = \mathrm{B}(\beta, \alpha, \gamma) \\ &= \mathrm{B}(\beta, \gamma, \alpha) = \mathrm{B}(\gamma, \alpha, \beta) = \mathrm{B}(\gamma, \beta, \alpha).\end{aligned} \tag{10}$$

(iii) Recursion

$$\mathrm{B}(\alpha+1, \beta, \gamma) = \frac{\alpha}{\alpha+\beta+\gamma}\mathrm{B}(\alpha, \beta, \gamma); \tag{11}$$

$$\mathrm{B}(\alpha, \beta+1, \gamma) = \frac{\beta}{\alpha+\beta+\gamma}\mathrm{B}(\alpha, \beta, \gamma); \tag{12}$$

$$\mathrm{B}(\alpha, \beta, \gamma+1) = \frac{\gamma}{\alpha+\beta+\gamma}\mathrm{B}(\alpha, \beta, \gamma). \tag{13}$$