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Option pricing under regime-switching jump–diffusion models

Massimo Costabile, Arturo Leccadito, Ivar Massabó, Emilio Russo *

Department of Economics, Statistics, and Finance, University of Calabria, Ponte Bucci cubo 3C, 87036, Rende (CS), Italy

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ABSTRACT

We present an explicit formula and a multinomial approach for pricing contingent claims under a regime-switching jump–diffusion model. The explicit formula, obtained as an expectation of Merton-type formulae for jump–diffusion processes, allows to compute the price of European options in the case of a two-regime economy with lognormal jumps, while the multinomial approach allows to accommodate an arbitrary number of regimes and a generic jump size distribution, and is suitable for pricing American-style options. The latter algorithm discretizes log-returns in each regime independently, starting from the highest volatility regime where a recombining multinomial lattice is established. In the remaining regimes, lattice nodes are the same but branching probabilities are adjusted. Derivative prices are computed by a backward induction scheme.

1. Introduction

Several empirical studies show that financial returns exhibit volatility with a stochastic pattern and fatter tails than the standard normal model, which has been shown not to be suitable for capturing the asset price dynamics. Consequently, many alternative approaches have been proposed in order to capture the dynamics of financial returns. Examples are the jump–diffusion process introduced by Merton [1], and the regime-switching model introduced by Hamilton [2,3] that has become more and more attractive for researchers who, starting from the first contribution presented by Naik [4] in 1993, have developed a wide range of option pricing models in this framework.

In this paper, we propose an explicit formula and a multinomial approach for evaluating contingent claims when the underlying asset dynamics evolves according to a regime-switching model with jumps. The choice of this framework is motivated by two main aspects:

- regime-switching models represent a simple way to capture stochastic volatility and, hence, fat tails, thus overcoming the drawback of the classical lognormality assumption characterized by constant volatility;
- the addition of a jump component to the regime-switching context contributes to explain accurately some of the empirical biases evidenced by the classical lognormal model.

In financial literature, regime-switching models have been prevalently applied in order to allow Lévy processes to switch in a finite state space. Among the contributions in the field of option pricing, it is worth mentioning Konikov and Madan [5], who introduce an extension of the variance-gamma model in which the parameters switch, according to a two-state Markov chain, between two fixed sets of values at infinitesimal time intervals. Furthermore, they evidence that more than two states should be considered for the Lévy process, but the mathematical approach they use cannot easily accommodate more than two states for option valuations. To overcome this limit, Elliot and Osakwe [6] extend their work to more than two

* Corresponding author. Tel.: +39 0984493387; fax: +39 0984492277.
E-mail addresses: massimo.costabile@unical.it (M. Costabile), arturo.leccadito@unical.it (A. Leccadito), i.massabo@unical.it (I. Massabó), emilio.russo@unical.it (E. Russo).

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states introducing a multi-state Markov switching model where the underlying process is a jump process with parameters that may switch among drift/compensator pairs. Albanese et al. [7] develop a model similar to the one of Konikov and Madan [5] except that switches occur only at finite time intervals, deriving as well closed form formulae for European options. Jackson et al. [8] propose a Fourier space time-stepping algorithm to derive option prices in a regime-switching Lévy process. Jiang and Pistorius [9] evaluate perpetual American put options in an exponential regime-switching Lévy model deriving analytically tractable results. A different approach, which uses a Markov chain with a progressively denser state space to approximate a continuous time stochastic volatility model with jumps, has been proposed by Chourdakis [10] who in this way obtains option prices in semi-closed form.

Among the contributions in option pricing that consider a regime-switching model with jumps, it is worth mentioning Yuen and Yang [11] who, after generalizing the Naik [4] model to more than two regimes, provide a trinomial lattice to price options under a jump–diffusion Markov regime-switching model. Indeed, as in Naik [4], the underlying asset process presents jumps only during the switches among states with the jump size depending upon the state before and after the switching and the current asset price. A more general framework is proposed in Ramponi [12], who presents a Fourier transform method to compute the price of European contingent claims when the underlying asset behavior is described by a jump–diffusion dynamics with parameters driven by a continuous time and stationary Markov chain on a finite state space. His method is suitable for pricing European-style contingent claims but may not be applicable to evaluate American options.

In this paper, we work in the framework proposed by Ramponi [12] where a regime-switching model for the underlying asset embedding a jump component that may switch among different regimes is considered. After a preliminary econometric analysis that supports the choice of a regime-switching jump–diffusion dynamics to model the equity price, we present an explicit formula to compute the price of European options in the case of a two-regime economy with jumps in the asset price process following a lognormal distribution. The formula is obtained as an expectation of Merton-type formulae for jump–diffusion processes by conditioning the asset distribution on the occupation time in one of the two regimes and on the number of jumps occurring in each regime. It does not require possibly cumbersome inversions and represents an alternative approach for option pricing with respect to the Ramponi's [12] formula, which is obtained applying Fourier methods. To complete the treatment of the pricing problem, we also propose a discrete multinomial approach that is flexible enough to accommodate an arbitrary distribution for the jump component and an arbitrary number of regimes both for the diffusion and the jump component, and presents the advantage of being easily applied to price American-style options. We develop a multinomial lattice which is needed to capture both the diffusion and the jump component in the underlying asset process associated to the highest volatility regime. Indeed, we approximate the diffusion part by a trinomial tree and add more branches to capture jumps. For the other regimes, instead of generating new lattices, we simply adjust branching probabilities as suggested by Yuen and Yang [13]. Then, option prices are computed via backward induction. Numerical results within a two-state regime-switching version of the Merton [1] jump–diffusion model are also provided to support the model.

The rest of the paper is organized as follows. After a preliminary section presenting the framework and the econometric analysis aimed at validating regime-switching models with jumps (Section 2), we develop the explicit formula to evaluate European options in the presence of a two-regime economy when jumps follow a lognormal distribution (Section 3), and the multinomial lattice for more general cases (Section 4). Section 5 presents numerical results confirming the accuracy of the proposed model for European, American, and barrier options, and provides a numerical discussion concerning the behavior of the option prices computed by the multinomial approach. Finally, Section 6 concludes.

2. Framework and econometric analysis

We divide this section into two parts. In the first one, we analyze the framework in which we will develop our model; then, in the second part, we provide an econometric analysis supporting the choice of the underlying framework. For the sake of simplicity, we limit the analysis to a Markov chain with only two states but its extension to a greater number of states is straightforward.

2.1. The framework

On a probability space $(\Omega, \mathcal{F}, \mathcal{P})$, where $\mathcal{P}$ is the real-world probability measure, we consider a risky asset in a security market where trading takes place in the interval $[0, T]$. The parameters of the asset dynamics may switch according to a continuous time, homogeneous and stationary Markov process, $\epsilon(t)$, on the state space $\mathcal{L} = \{0, 1\}$ with generator $A \in \mathbb{R}^{2\times 2}$,

$$A = \begin{pmatrix} -a_{0,1} & a_{0,1} \\ a_{1,0} & -a_{1,0} \end{pmatrix}, \tag{1}$$

governing the transition probabilities of the process from the current state to the other. The transition probability matrix in the interval $[t, t + \Delta t]$ is given by

$$P = e^{A\Delta t} = \sum_{n=0}^{\infty} \frac{(A\Delta t)^n}{n!} = I + A\Delta t + o(\Delta t),$$