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An iterative updating method for damped structural systems using symmetric eigenstructure assignment[☆]

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ABSTRACT

An iterative method for updating finite element models with measured modal results using a symmetric eigenstructure assignment technique is developed. By the method, the updated symmetric damping and stiffness matrices can be obtained within finite iteration steps in the absence of roundoff errors by choosing a special kind of initial matrices and the measured data are embedded in the updated model. The numerical results show that the proposed method is reliable and attractive.

1. Introduction

Vibrating structures with feedback controls, such as bridges, buildings, airplanes and automobiles, generated by finite-element methods, are most often modeled in a system of second-order differential equations:

$$M_a\ddot{q}(t) + D_a\dot{q}(t) + K_a q(t) = Bu(t), \tag{1}$$

where M_a, D_a and K_a are $n \times n$ symmetric matrices with M_a being nonsingular, which represent the analytical mass, damping and stiffness matrices, respectively. The time-dependent variable $q(t) \in \mathbf{R}^{n\times 1}$ is the position vector, $B \in \mathbf{R}^{n\times m}$ is the full rank control feedback matrix and $u(t) \in \mathbf{R}^{m\times 1}$ is the control vector. In addition, the output or measurement vector $y(t) \in \mathbf{R}^{r\times 1}$ is given by

$$y(t) = Cq(t), \tag{2}$$

where C is a real $r \times n$ output matrix. In discussing the feedback control, we usually assume that the control vector $u(t)$ is defined by the control law

$$u(t) = Fy(t) + G\dot{y}(t), \tag{3}$$

where $F, G \in \mathbf{R}^{m\times r}$ are output feedback gain matrices. Substituting (2) and (3) into (1) yields the following closed-loop system:

$$M_a\ddot{q}(t) + (D_a - BGC)\dot{q}(t) + (K_a - BFC)q(t) = 0. \tag{4}$$

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In order to reduce the numbers of design parameters, it is desirable to set $C = B^\top$. Namely, the input and output devices are placed at the same location. To solve the homogeneous second-order system Eq. (4) with $C = B^\top$ is known as to solve the quadratic eigenvalue problem (QEP)

$$(\lambda^2 M_a + \lambda(D_a - BGB^\top) + K_a - BFB^\top)x = 0 \tag{5}$$

by letting $q(t) = xe^{\lambda t}$ in Eq. (4), where $\lambda \in \mathbf{C}$ and $x \in \mathbf{C}^{n\times 1}$ are eigenvalues and eigenvectors of QEP, respectively. It is known that the equation of (5) has $2n$ finite eigenvalues over the complex field, provided that the leading matrix coefficient M_a is nonsingular. Note that the signification of the system (4) usually can be interpreted via the eigenvalues and eigenvectors of Eq. (5). Because of this connection, a lot of efforts have been devoted to the QEP in the literature. Many applications, properties and numerical methods for the QEP are surveyed in the thesis by Tisseur and Meerbergen [1].

Finite element (FE) models are widely used to predict the dynamic characteristics of structures and these models are constructed on the basis of highly idealized engineering blueprints and designs that may not truly represent all the physical aspects of the actual structures. These models often give results that differ from the measured results and therefore need to be updated to match the measured data. FE model updating entails tuning the model so that it can better reflect the measured data from the physical structure being modeled [2]. The problem of how to modify the analytical model from the dynamic measurements is known as the model updating in structural dynamics. Basically, FE model updating is an inverse problem to identify and correct uncertain parameters of FE models and it is usually posed as an optimization problem. The updated model may then be considered a better dynamical representation of the structure and used with greater confidence for the analysis of the structure under different boundary conditions or with physical structural changes.

In the past 30 years, various techniques for updating mass and stiffness matrices for undamped systems using measured response data have been discussed by Baruch [3], Baruch and Bar-Itzhack [4], Berman [5], Berman and Nagy [6], Wei [7–9], Yang et al. [10], Yang and Chen [11], and Yuan [12]. For damped structured systems, the theory and computation have been considered by Friswell et al. [13], Pilkey [14], Kuo et al. [15], Chu et al. [16] and Yuan [17]. The finite element model correction of the closed-loop system (4) using a symmetric eigenstructure assignment was proposed in [18,19]. The method incorporates the measured modal data into the finite element model to produce an adjusted finite element model on damping and stiffness with symmetric low-rank updating that matches the experimental modal data.

In vibration industries, through vibration tests where the excitation and the response of the structure at selected points are measured experimentally, there are identification techniques to extract a portion of eigenpair information from the measurements. However, quantities related to high frequency terms in a finite-dimensional model generally are susceptible to measurement errors due to the finite bandwidth of measuring devices. It is simply unwise to use experimental values of high natural frequencies to reconstruct a model. In fact, in a large and complicated physical system, it is often impossible to acquire knowledge of the entire spectral information. While there is no reasonable analytical tool available to evaluate the entire spectral information, we can attain only partial information through experiments. For this reason, it might be more sensible to consider a model updating using only a few measured eigenvalues and eigenvectors [2,6]. However, in practice, the eigenvectors are measured only at limited degrees of freedom due to hardware limitations. There are ways to deal with incomplete measured data, such as model reduction and model expansion techniques [2]. For the purpose of this paper, we will assume that the eigenvectors have been measured to the full degree of freedom or some measures have been taken so that a comparison with analytical eigenvectors is possible.

The problem of updating damping and stiffness matrices using symmetric eigenstructure assignment can be stated as follows.

Problem P. Let $B \in \mathbf{R}^{n\times m}$ be a full column rank matrix and $\Lambda = \mathrm{diag}\{\lambda_1, \ldots, \lambda_p\} \in \mathbf{C}^{p\times p}$, $X = [x_1, \ldots, x_p] \in \mathbf{C}^{n\times p}$ be the measured eigenvalue and eigenvector matrices, where $p < n$ and both Λ and X are closed under complex conjugation in the sense that $\lambda_{2j} = \bar{\lambda}_{2j-1} \in \mathbf{C}$, $x_{2j} = \bar{x}_{2j-1} \in \mathbf{C}^{n\times 1}$ for $j = 1, \ldots, l$, and $\lambda_k \in \mathbf{R}$, $x_k \in \mathbf{R}^{n\times 1}$ for $k = 2l+1, \ldots, p$. Find $(\hat{G}, \hat{F}) \in \mathbf{S_E}$ such that

$$\|B\hat{G}B^\top\|^2 + \|B\hat{F}B^\top\|^2 = \min_{(G,F)\in\mathbf{S_E}} (\|BGB^\top\|^2 + \|BFB^\top\|^2),$$

where

$$\mathbf{S_E} = \{(G, F) \in \mathbf{SR}^{m\times m} \times \mathbf{SR}^{m\times m} | M_a X\Lambda^2 + (D_a - BGB^\top)X\Lambda + (K_a - BFB^\top)X = 0\}.$$

Note that the studies in the works by Chu and Datta [20], Datta [21], Nichols and Kautsky [22], Datta et al. [23], Datta and Sarkissian [24] and Lin and Wang [25] lead to a feedback design problem for a second-order control system. That consideration eventually results in either a full or a partial eigenstructure assignment problem for the QEP. The proportional and derivative state feedback controller designated in these studies is capable of assigning specific eigenvalues and making the resulting system insensitive to perturbations. Nonetheless, these results cannot meet the basic requirement that the updated matrices should be symmetric. Recently, Kuo, Lin and Xu [26] have presented a direct correcting method for quadratic eigenvalue problems using symmetric eigenstructure assignment and it seems that the algorithm proposed is reliable and attractive. However, we observe that in order to use this method, one must solve the system of linear equation (40) of [26], which involves in intricate matrix computations. In view of the complicated representation of solutions for the direct updating method, our main contribution in this paper is to provide an alternative iterative method to solve Problem P.