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On solvability of linear systems generated by the complex variable boundary element method

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ABSTRACT

Within the complex variable boundary element method, an approximate solution is determined by a Cauchy-type integral whose density is a piecewise linear function. Such an integral can be expressed by a linear combination of some functions that can be chosen in many ways. The choice influences properties of a linear system that arises by discretization of some boundary value problem. One choice is presented that allows to deduce some results about the system solvability. It is demonstrated on the Dirichlet problem.

1. Introduction

From the middle of the 1980s, T.V. Hromadka II and his cooperators develop a new method, the complex variable boundary element method, shortly CVBEM, see for example [1,2] etc., allowing to approximately solve some planar boundary value problems. The main goal of the CVBEM is the fact that any Cauchy-type integral can be approximated by another one whose density is a suitable piecewise linear function. The Cauchy-type integral having a piecewise linear density can be expressed by a linear combination

$$\sum_{j=0}^{m-1} l_j f_j, \tag{1}$$

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of some functions $f_0, \dots, f_{m-1}$ by different ways. If these functions are suitably chosen and if the coefficients $l_0, \dots, l_{m-1}$ are enumerated from a system of linear equations that reflect prescribed boundary conditions, then (1) can generate an approximate solution of the appropriate boundary value problem. The choice of the functions $f_0, \dots, f_{m-1}$ influences many aspects of the CVBEM numerical analysis, e.g. the solvability of the appropriate linear system. There have been presented large possibilities of how to choose the functions $f_0, \dots, f_{m-1}$ in [3–5] etc. However, without any supporting results on the solvability.

In this article we present such a choice of the functions $f_0, \dots, f_{m-1}$ that we can prove two statements on the appropriate linear system solvability. In Sections 2 and 3 we define some terms related to region boundaries and boundary functions. In Section 4 we show some properties of Cauchy-type integrals having a piecewise linear density. We introduce the functions $f_0, \dots, f_{m-1}$ and prove that their real parts $g_0, \dots, g_{m-1}$ are linearly independent in Section 5; this is the crucial property allowing to derive the two main statements in Section 7. In Section 6 we use the functions $g_0, \dots, g_{m-1}$ to construct an approximate solution of the Dirichlet problem.

2. Paths

Let $-\infty < \alpha < \beta < +\infty$. A continuous mapping

$$\gamma : \langle \alpha, \beta \rangle \longrightarrow \mathbb{C} \tag{2}$$

we call a *path* if there exist $n \in \mathbb{N}$ and a sequence $\{\tau_j\}_{j=0}^{n}$ such that

$$\alpha = \tau_0 < \tau_1 < \cdots < \tau_n = \beta$$

and that the derivative γ' of the mapping γ is continuous and nonzero in every interval $\langle \tau_j, \tau_{j+1} \rangle$, where $j \in \{0, \dots, n-1\}$. We say that the path (2) *starts* at the point $a = \gamma(\alpha)$ and *ends* at the point $b = \gamma(\beta)$; we set

$$[\gamma] = \{\gamma(t) : \alpha \le t \le \beta\}.$$

By the symbol $\dot{-}\gamma$ we denote the path *opposite* to the path γ. Then $[\dot{-}\gamma] = [\gamma]$. If a path γ_1 starts at the same point as another path γ_0 ends, then $\gamma_0 \vee \gamma_1$ denotes the path emerged by connection of the paths γ_0, γ_1. Then $[\gamma_0 \vee \gamma_1] = [\gamma_0] \cup [\gamma_1]$. The path (2) is called a *simple arc* if the mapping γ is injective. If $\gamma(\alpha) = \gamma(\beta)$, then γ is a *closed* path. γ is a *Jordan* path if it is closed and

$$0 < |t - t'| < \beta - \alpha \Longrightarrow \gamma(t) \neq \gamma(t')$$

for every $t, t' \in \mathbb{R}$.

In this work, the notation (F, D) represents a *holomorphic element*, i.e. a function F holomorphic in a region $D \subset \mathbb{C}$.

We consider the path (2), $k \in \mathbb{N}$, a sequence $\{s_i\}_{i=0}^{k+1}$ such that

$$\alpha = s_0 < s_1 < \cdots < s_k = s_{k+1} = \beta$$

and a sequence $\{(F_i, D_i)\}_{i=0}^{k}$ such that D_i is an open circle containing the set $\gamma(\langle s_i, s_{i+1} \rangle)$ for every $i \in \{0, \dots, k\}$ and that

$$F_i(z) = F_{i+1}(z), \quad z \in D_i \cap D_{i+1},$$

for every $i \in \{0, \dots, k-1\}$. Further, we consider holomorphic elements (F, D), (F^*, D^*) such that

$$D_0 \subset D, \qquad D_k \subset D^*, \qquad F_0 = F|_{D_0}, \qquad F_k = F^*|_{D_k}.$$

The considered situation means that (F^*, D^*) is the *analytic continuation* of (F, D) along the path γ, which we describe by the schema

$$(F, D) \xrightarrow{\gamma} (F^*, D^*).$$

We set

$$\gamma[z] = \int_\gamma \frac{\mathrm{d}\zeta}{\zeta - z}, \quad z \in \mathbb{C} \setminus [\gamma].$$

Then $\gamma[\cdot]$ presents a holomorphic function with the derivative

$$\frac{\mathrm{d}}{\mathrm{d}z}\gamma[z] = \frac{1}{z-b} - \frac{1}{z-a}, \quad z \in \mathbb{C} \setminus [\gamma]. \tag{3}$$

Certainly,

$$(\dot{-}\gamma)[z] = -\gamma[z], \quad z \in \mathbb{C} \setminus [\gamma],$$
$$(\gamma_0 \vee \gamma_1)[z] = \gamma_0[z] + \gamma_1[z], \quad z \in \mathbb{C} \setminus [\gamma_0 \vee \gamma_1].$$

If γ is a Jordan path, then we denote by Ω_γ the bounded component of the open set $\mathbb{C} \setminus [\gamma]$ in the sense of the Jordan theorem. A Jordan path γ is *positively oriented* if

$$z \in \Omega_\gamma \Longleftrightarrow \gamma[z] = 2\pi i.$$