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Fast linearized alternating direction minimization algorithm with adaptive parameter selection for multiplicative noise removal

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ABSTRACT

Owing to the edge preserving ability and low computational cost of the total variation (TV), variational models with the TV regularizer have been widely investigated in the field of multiplicative noise removal. The key points of the successful application of these models lie in: the optimal selection of the regularization parameter which balances the data-fidelity term with the TV regularizer, the efficient algorithm to compute the solution. In this paper, we propose two fast algorithms based on the linearized technique, which are able to estimate the regularization parameter and recover the image simultaneously. In the iteration step of the proposed algorithms, the regularization parameter is adjusted by a special discrepancy function defined for multiplicative noise. The convergence properties of the proposed algorithms are proved under certain conditions, and numerical experiments demonstrate that the proposed algorithms overall outperform some state-of-the-art methods in the PSNR values and computational time.

1. Introduction

Images contaminated by the additive Gaussian noise are widely investigated in the field of image restoration. However, for coherent imaging systems such as laser or ultrasound imaging, synthetic aperture radar (SAR) and optical coherence tomography, image acquisition processes are different from the usual optical imaging, and thus the multiplicative noise (speckle), rather than the additive noise, provides an appropriate description of these imaging systems. In this paper, we mainly consider the problem of the multiplicative noise removal.

Let $f \in \mathbb{R}^{m\times n}$ be the observed image corrupted by the multiplicative noise η. Our goal is to recover the original image $u \in \mathbb{R}^{m\times n}$ from the observed image. The corresponding relationship can be expressed as follows:

$$f = u \cdot \eta. \tag{1.1}$$

In the SAR system, the noise follows a negative exponential law, and hence the observed image is degraded by the noise very seriously. The conventional SAR system reduces the effect of noise by the multi-looking process, i.e., averaging the observed images from slightly different angles of the same resolution cell (M-look SAR image). Therefore, the probability

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density function of the noise η follows a Gamma distribution:

$$P_\eta(x) = \frac{M^M}{\Gamma(M)} x^{M-1} e^{-Mx} 1_{\{x \geq 0\}}, \tag{1.2}$$

where M is the number of looks. The mean value of the speckle η is 1, and the variance of η is $\frac{1}{M}$.

In the last several years, various variational models and iterative algorithms [1–7] based on the TV regularization have been proposed for the multiplicative noise removal. Aubert and Aujol [1] used a maximum a posteriori (MAP) estimator and established the following TV regularized variational model:

$$\min_u \tau \langle \log u + fu^{-1}, \mathbf{1} \rangle + \|\nabla u\|_1, \tag{1.3}$$

where $\tau > 0$ is a fixed regularization parameter, and $\|\nabla u\|_1$ is the (isotropic) total variation of u, i.e.,

$$\|\nabla u\|_1 = \sum_{i=1}^{m} \sum_{j=1}^{n} \sqrt{\left(\nabla_{ij}^1 u\right)^2 + \left(\nabla_{ij}^2 u\right)^2}.$$

Note that $\nabla_{ij}^1 u$ and $\nabla_{ij}^2 u$ denote the horizontal and vertical first-order differences at pixel (i, j) respectively, $\mathbf{1}$ denotes a matrix of ones with the same size as u, the multiplication and division are performed in componentwise, and Neumann boundary conditions are used for the computation of the gradient operator ∇ and its adjoint (just the negative divergence operator) $-\mathrm{div}$; see [7,8] for more details.

The objective function in the AA model (1.3) is nonconvex, and hence it is difficult to find a global optimal solution. Recently, in many existing literatures [2,3], the log transformation was used to resolve the non-convexity. The variational model based on the log transformed image can be reformulated as follows:

$$\min_u \tau \langle u + fe^{-u}, \mathbf{1} \rangle + \|\nabla u\|_1. \tag{1.4}$$

The above model overcomes the drawback of the AA model, and numerical experiments verify its efficiency [2]. In [4], the exponential of the solution of the exponential model (1.4) is proved to be equal to the solution of the classical I-divergence model

$$\min_u \tau \langle u - f \log u, \mathbf{1} \rangle + \|\nabla u\|_1. \tag{1.5}$$

Very recently, new convex methods such as shifting technique [6] and m-th root transformation [9] were also investigated for this problem.

The augmented Lagrangian framework has recently been proposed to solve the exponential model (1.4) and the I-divergence model (1.5). Although this iterative technique is useful, inner iteration [3] or inverses involving the Laplacian operator [4] are required at each iteration. The computational cost of the inner iteration or the inverse operation is still expensive. Very recently, linearized techniques [10–13] were widely used for accelerating the alternating minimization algorithm for solving the variational models in image processing. In [14], a fast proximal linearized alternating direction (PLAD) algorithm, which linearized both the fidelity term and the quadratic term of the augmented Lagrangian function, was proposed to solve the TV models (1.4) and (1.5). Numerical results there demonstrate that the PLAD algorithm overall outperforms the augmented Lagrangian method for multiplicative noise removal.

The regularization parameter τ in (1.4) controls the trade-off between the goodness-of-fit of f and a smoothness requirement due to the TV regularization. The regularized solution highly depends on the selection of τ. Specifically, large τ leads to little smoothing, and thus, noise will remain almost unchanged in the denoised image or the regularized solution, whereas small τ leads to oversmoothing solution, so that fine details in the image are destroyed. Therefore, choosing a suitable τ is a key issue for the variational model (1.4). In the existing literatures, the main approaches for automatic parameter setting include the generalized cross validation [15], the L-curve [16,17], the Stein unbiased risk estimator [18] and the discrepancy principle [19]. These methods are all based on the assumption of Gaussian noise and not suitable for the special denoised problem here. In [7], a new discrepancy principle based on the statistical characteristics of the multiplicative noise was proposed for selecting a proper value of τ. However, it needs to solve (1.4) or the corresponding I-divergence model several times for a sequence of τ's. Hence the computational cost is expensive.

In this paper, we use the special discrepancy principle for the multiplicative noise to compute τ. A linearized alternating direction minimization algorithm with auto-adjusting of the regularization parameter is proposed to solve (1.4). In each iteration of the proposed algorithm, the regularization parameter is updated in order to guarantee that the denoised image satisfies the discrepancy principle. Due to the use of the linearization technique, the solution of the subproblem in the iteration step has a closed-form expression, and therefore the zero point of the discrepancy function with respect to τ can be computed by the Newton method efficiently. Numerical experiments show that our algorithm is very effective in finding a proper τ. Moreover, the proposed algorithm can be extended to the case of the spatially-adapted regularization parameter without extra computational burden almost.

We will give the convergence proof of the proposed algorithms with adaptive parameter estimation. With a fixed regularization parameter, our algorithms are reduced to the PLAD algorithm proposed in [14]. As we know, the convergence