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Application of the collocation method for solving nonlinear fractional integro-differential equations

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ABSTRACT

In this paper, using the collocation method we solve the nonlinear fractional integro-differential equations (NFIDE) of the form:

$$\begin{aligned}
&f(t, y(t), {}_a^C D_t^{\alpha_0} y(t), \ldots, {}_a^C D_t^{\alpha_r} y(t)) = \lambda G\left(t, y(t), \int_a^t k(t,s)F(s, y(s))ds\right),\\
&y^{(k)}(a) = d_k,\\
&k = 0, 1, \ldots, m_0 - 1.
\end{aligned}$$

We study the convergence and the stability analysis of this method for $f(t, y(t), {}_a^C D_t^{\alpha_0} y(t), \ldots, {}_a^C D_t^{\alpha_r} y(t)) = y(t) + \sum_{j=0}^{r} b_j\, {}_a^C D_t^{\alpha_j} y(t) + g(t)$. Some numerical examples are given to show the efficiency of the presented method.

1. Introduction

1.1. The applications

Fractional differential equations and fractional integro-differential equations are used in modeling of many physical and chemical processes. For example in [1] authors developed a mathematical model for a micro-electro-mechanical system (MEMS) instrument that has been designed to measure the viscosity of fluids that are encountered during oil well exploration. The device may be used in two fashions, in one of them the device is released from an initial displacement and its subsequent decaying oscillations are measured. The displacement of the device satisfies a fractional differential equation of the form:

$$\begin{aligned}
&{}_a^C D_t^2 x(t) + \beta\sqrt{\pi}\,{}_a^C D_t^{\frac{1}{2}} x(t) + \alpha\,{}_a^C D_t^1 x(t) + x(t) = 0,\\
&x(0) = 1,\\
&x'(0) = 0,
\end{aligned}$$

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where α and β are constant. Also the process of the solution of a gas in a fluid is described by a fractional differential equation of the form [2]:

$$F(t)\,{}_a^C D_t^1 x(t) + G(t)\,{}_a^C D_t^{\frac{1}{2}} x(t) + x(t) = -1.$$

Also we consider the fractional differential equation of the form:

$$m\,{}_a^C D_t^2 x(t) + c\,{}_a^C D_t^{\frac{1}{2}} x(t) + kx(t) = AF_0(t),$$

which describes the mass moving about a guide with a known force F_0, opposed by a viscoelastic damping force and a spring force where c, k and m are the damping, spring coefficients and the mass of the block, respectively. When the system undergoes pure viscoelastic friction at high-speeds and a viscous friction at low-speeds, it is said to be viscoelastic–viscous. If the behavior is opposite, it is called viscous–viscoelastic. The fractional differential equation that describes this mechanism is:

$${}_a^C D_t^2 y(t) + c_0 f(y(t))\,{}_a^C D_t^1 y(t) + c_0(1 - f(y(t)))\,{}_a^C D_t^{\frac{1}{2}} y(t) + k_0 y(t) = F_0^*(t),$$

where

$$f(y(t)) = \begin{cases} y^2(t) & \text{if viscous–viscoelastic,} \\ 1 - y^2(t) & \text{if viscoelastic–viscous.} \end{cases}$$

Also c_0 and k_0 are constant. The distance parameter, x, is defined for $x = \pm L$ and $y = \frac{x}{L}$. For more applications of the fractional differential equations [3–5] in relaxation processes and reaction kinetics of proteins, dynamic problems of solid mechanics, modeling the cardiac tissue electrode interface and signal processing, one can see [6–11], respectively.

1.2. Literature review

There have been proposed different numerical techniques for solving fractional differential equations and fractional integro-differential equations. For example in [12] E.A. Rawashdeh studied the numerical solution of an integro-differential equation with the fractional derivative of the type:

$$\begin{aligned} &{}_a^C D_t^\alpha y(t) = p(t)y(t) + f(t) + \int_0^t k(t,s)y(s)ds, \\ &y(0) = a, \end{aligned}$$

by polynomial spline functions. The authors of [13] proposed a new method based on the Taylor collocation method [14] and obtained the approximate solution of linear fractional differential equation with variable coefficients. Also in [15], authors presented an analytical solution for the fractional integro-differential equation of the type:

$${}_0^C D_t^\alpha y(t) = p(t)y(t) + f(t) + \int_0^t k(t,s)y(s)ds, \quad 0 < \alpha < 1, t \in [0,1], \qquad y(0) = a.$$

In [16] authors used the Adomian decomposition method for solving

$${}_0^C D_t^\alpha y(t) = p(t)y(t) + f(t) + \int_0^t k(t,s)F(y(s))ds, \quad 0 < \alpha < 1, t \in [0,1], \qquad y(0) = a.$$

In [17] authors investigated the solvability of the following multi-term fractional differential equation in a reflexive Banach space:

$$\sum_{j=0}^r b_j\,{}_0^C D_t^{\alpha_j} y(t) = f(t, y(t)), \quad 0 < \alpha < 1, t \in [0,1], \qquad y(0) = 0.$$

In [18] Dubois and Mengué employed the mixed collocation method for solving the following equation:

$${}_a^C D_t^\alpha y(t) = f(t, y(t)), \quad 0 < \alpha < 1.$$

Furthermore authors of [19] presented a shifted Chebyshev collocation method which uses the shifted Chebyshev–Gauss points as collocation nodes for solving the nonlinear multi-order fractional initial value problems. The authors of [20] studied the pseudo-spectral method [21] for solving fractional differential equations of the form:

$$\begin{aligned} &a\,{}_a^C D_t^\alpha y(t) + b\,{}_a^C D_t^\beta y(t) + cy(t) = g(t), \\ &y(0) = c_0, \\ &y'(0) = c_1, \end{aligned}$$

where $t \in [0, T]$ and $0 \le \beta < \alpha \le 2$. The author of [22] proposed an implicit algorithm for the approximate solution of a class of fractional differential equations. Also authors of [23] obtained an approximate solution for nonlinear fractional differential equations by spline collocation methods.